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New Geometry Exhibit at Chicago's Museum of Science and Industry

(see page 263)

EDWIN W. SCHREIBER

Secretary-Treasurer of
The National Council of Teachers of Mathematics, 1929–51
and Member of the Board of Directors, 1928–29

THE MATHEMATICS TEACHER

Volume XLV Number 4

Edwin W. Schreiber
1890-1951

By H. GLENN AYRE
Western Illinois State College, Macomb, Illinois

"RASCH tritt der Tod den Menschen an, es ist ihm keine Frist gegeben."—Death comes to mankind without warning; he is given no respite. Thus wrote Edwin W. Schreiber in a letter to a boyhood friend just a few days before death came to him without warning and without respite.

As faculty and students were assembling for the regular college day on the morning of December 17, 1951, they were shocked and saddened on receiving word of the sudden death of Professor Schreiber. He had been a member of the Mathematics Department since 1929 and had served as acting head of the department since the retirement of Professor R. M. Ginnings in 1941. In addition to being a careful and thorough classroom teacher he served Western in such capacities as Secretary-Treasurer of the Athletic Association, Chairman of the Faculty Club, and performed official duties on numerous committees. His hobby of photography led to the production of many movies and slides of special events, such as homecoming parades, football games, commencement processions, and the like.

Mr. Schreiber was born at Saginaw, Michigan, January 20, 1890, son of Hugo and Adelaide Reuter Schreiber. He was reared in Saginaw and graduated from the Arthur Hill High School of that city in 1908. He received the Bachelor's degree from the University of Michigan in 1911, and the Master's degree from the University of Chicago in 1924. He also studied at the University of California at Berkeley and pursued advanced graduate study at the University of Michigan for two years, 1927–29, where his greatest interest was in the field of the history of mathematics.

On August 28, 1916, he married Elizabeth Patterson, who survives with a son, Robert Schreiber, of the faculty of the University of Maine, at Orono, Maine; a granddaughter, Sherry Sue; and a brother Dr. Carl F. Schreiber, head of the German Department of the Sheffield Scientific School at Yale University.

His record as an educator is an impressive one; his experience was wide and varied. He was head of the mathematics department at the high school at Yuma, Arizona, for two years, head of mathematics at the high school, New Castle, Pennsylvania, for three years, and head of mathematics at Proviso Township High School, Maywood, Illinois, for ten years, 1917–1927.

Mr. Schreiber took a keen interest in professional organizations and gave unselfishly of his time and effort to promote the improvement of the teaching of mathematics. One can hardly think of the National Council of Teachers of Mathematics without associating with it the

name of Schreiber. He was a charter member, a member of the Board of Directors from 1928 to 1929, and served as its second Secretary-Treasurer from 1929 to 1951. He appeared frequently on the programs at annual meetings of the council and served on many committees of national scope. His outstanding contribution to the growth and development of the organization is difficult to estimate and can scarcely be duplicated. He was one of the foremost leaders in the Central Association of Science and Mathematics Teachers. He joined the organization in 1918, served two three-year terms on the Board of Directors—1930–31–32, and again in 1940–41–42. He was Vice-President of the Association in 1943, chairman of the Mathematics Section in 1928, and presented many papers before this Section. In 1943 the Association created a new office, that of Historian, and appointed Mr. Schreiber the first official historian. He was also a charter member of the History of Science Society and served on its committees; a member of the Mathematical Association of America; the Men's Mathematics Club of Chicago, serving as President in 1924; chairman of the Mathematics Section of the Illinois High School Conference, 1924, and later a member of the subsequent organization, the Illinois Council of Teachers of Mathematics. His scholarly attainments were recognized by the American Association for the Advancement of Science when he was honored by being elected a Fellow of that organization in 1924.

Mr. Schreiber served these organizations generously and with sincerity of purpose, always deeming it a high honor to work with other outstanding leaders of the nation in the promotion of mathematics education.

He contributed many articles to mathematics education including one on the "History of the Metric System" and one on the "History of Elementary Mathematical Instruments." He made a most worthy contribution to *A Half Century of Science and Mathematics Teaching,* a recent publication of the Central Association of Science and Mathematics Teachers. Other articles were published in The Mathematics Teacher and *School Science and Mathematics.*

In the local community Mr. Schreiber displayed the same urge to be of service as he did in professional organizations. He was a member of the Masonic Lodge, and of Kiwanis in which he held office as past president and as historian. He was a lifelong Presbyterian and served the church as Sunday School teacher, general superintendent and as an Elder for more than thirty years. A thread of his philosophy of life may be gleaned from a recent statement to a friend who had just remarked, "I recall that Kepler once said of his discoveries in astronomy: 'I am but thinking God's thoughts after him'." Then came the comment from Mr. Schreiber: "That is the way I feel about my work. You know, I think God must be somewhat of a Mathematician, there is so much of order and precision in the universe." It is fitting and proper to say that Edwin W. Schreiber was a man of deep religious convictions, high ideals, and unquestionable integrity. We have lost a true friend and a gentle life; the cause of humanity and education have suffered irreparably.

Symposium on Teacher Education in Mathematics

The Mathematical Association of America and the National Council of Teachers of Mathematics are jointly presenting a **"Symposium on Teacher Education in Mathematics."** The time and place are **Madison, Wisconsin, August 26–30, 1952,** inclusive. Persons attending the symposium will be lodged in student housing of the University at reasonable rates. There will be accommodations for families. Everyone interested in receiving further announcements of this symposium is invited to write to the director, Rudolph E. Langer, 822 Miami Pass, Madison 5, Wisconsin.

The President's Page

SPECIALIST FOR MATHEMATICS U. S. OFFICE OF EDUCATION

We are happy to announce that the position of Specialist for Mathematics in the U. S. Office of Education has been created and that Dr. Kenneth E. Brown, one of our leading members of the National Council, is its first incumbent. Dr. Brown assumed the duties of this new position on January 28 of this year.

For several years the National Council has been very much concerned that mathematics has not been represented in the U. S. Office of Education. We have been quite active in our efforts to get this position established. Since becoming a Department of the NEA, we have enjoyed closer association with the U. S. Office of Education and have been particularly active in this matter of getting a Specialist for Mathematics appointed.

In harmony with the desire of the National Council of Teachers of Mathematics and other professional organizations interested in mathematics education, the U. S. Office of Education has created the position of Specialist for Mathematics. The creation of this position is a timely one in light of the need for better training in mathematics for our youth in this atomic age. Members of the National Council will undoubtedly be anxious to make use of this new service of the U. S. Office of Education.

The services of the Specialist for Mathematics are available to groups of teachers of mathematics and to state departments of education. Services are also available for the improvement of mathematics curricula through workshops, institutes, and conferences sponsored by teachers and their state departments of education.

Members of the National Council will recall that for several years Dr. Brown

Dr. Kenneth E. Brown

was Chairman of the State Representatives. Dr. Brown's active participation in the work of the National Council should encourage close cooperation between the National Council and the U. S. Office of Education.

Dr. Brown's wide range of teaching experience should provide him with a valuable background as a consultant in mathematics education. His teaching experience, which ranges from elementary through college, has prepared him to meet the problems in the teaching of mathematics at all grade levels. His college teaching experience has been at the following institutions: Colorado State College of Education, Greeley; New Jersey State Teachers College, Paterson; Adelphi College, Long Island; East Carolina State College, Greenville, North Carolina; Wagner College, Staten Island; University of California at Los Angeles; University of Okla-

homa, Norman; and the University of Tennessee, Knoxville.

During the past three years Dr. Brown has been employed in the Mathematics Department of the University of Tennessee as Mathematics Counselor. He has worked directly with public school teachers in their classrooms in trying to help them improve their techniques of teaching and in exploring better ways of teaching mathematics. He has worked with city and county groups of teachers in conferences and workshops throughout the state on improving their mathematics programs.

Dr. Brown, a native of Oklahoma, received the Bachelor of Science degree from Oklahoma State College at Edmond in 1935; the Master of Arts degree from Colorado State College of Education at Greeley in 1937; and the degree of Doctor of Philosophy from Teachers College, Columbia University at New York City in 1942.

In addition to a well rounded training in mathematics education, Dr. Brown has had valuable experience in industry. His experience as general construction foreman, member of the engineering personnel for Sperry Gyroscope Company, and Electronics Specialist in the Navy, has shown him many meaningful applications of mathematics. His duties in the Navy permitted him to travel in South America, Africa and the British Isles.

In 1949 Dr. Brown married Margaret L. Berger who was then teaching mathematics at the University of Alabama. They have one child, Larry Kenneth, who is only a few months old. However, at four and a half months of age he attended the 1951 Christmas Meeting of the National Council at Stillwater, Oklahoma. Dr. Brown is a member of The Mathematical Association of America, but he says that he is not going to insist that Larry attend the meetings of this organization for a year or so.

Dr. Brown's primary interest has always been in the field of mathematics education, yet he has tried to keep a well rounded social life. This is evidenced by the fact that he was presented a key for his activities in the Lions Club and he is a member of the following fraternities, Lamda Sigma Tau, Kappa Delta Pi, and Phi Delta Kappa.

Dr. Brown has written the book, "General Mathematics in American Colleges," and he has contributed articles to THE MATHEMATICS TEACHER on general mathematics and geometry. His biographical sketch is included in the 1949 issue of "American Men of Science" and the 1947–48 issue of "Who's Who in Education."

We extend to Dr. Brown our hearty congratulations and best wishes. Also, we wish to assure him of the complete cooperation of the National Council in his efforts to give services to mathematics teachers and in his endeavors to aid in the improvement of mathematics education.

If you wish to write to Dr. Brown, address him at the U. S. Office of Education, Federal Security Building, Washington 25, D. C. I am sure that he will be glad to hear from you.

H. W. CHARLESWORTH
President

May Expirations

If the May issue of THE MATHEMATICS TEACHER is the last one you are to receive until your membership is renewed, **please send in your renewal promptly.** Postponing your renewal often means forgetting it, resulting in missed or delayed issues. The critical problems existing today because of the shortage of technical manpower and the defense effort emphasize the basic importance of mathematics to the modern age. At this time it is especially important that teachers of mathematics remain alert and the Council be kept strong.—M. H. AHRENDT, *Executive Secretary*

Notes on the Affiliated Groups

JOHN R. MAYOR, *Chairman*
Committee on Affiliated Groups
University of Wisconsin, Madison, Wisconsin

NEWSLETTERS OF THE AFFILIATED GROUPS

IN THE fall number of the Newsletter of Affiliated Groups information on bulletins of the various Affiliated Groups was given so that the records on these publications could be kept up-to-date. Officers of the Groups were asked to keep the Committee informed about their present policies and plans for publications. These publications have become an important factor in the growth and development of many of the affiliated organizations.

Most of the Affiliated Groups bulletins are mimeographed. They vary in size from two pages to ten. The usual number of issues per school year is four but there is some variation from this. Three of the better known of these publications have appeared as printed publications for some time. These are from Kansas, California and New Jersey. The first number of Volume 26 of the *Bulletin of the Kansas Association of Teachers of Mathematics* has been sent to members this fall. This year, Volume 9 of *The California Mathematics Council Bulletin*, and Volume 8 of *The New Jersey Mathematics Teacher* are in publication.

Financial statements which would show costs of publication and profit or loss and statements about methods of obtaining copy and of getting the bulletins published would be very useful for the Committee to have to distribute to other interested Affiliated Groups.

Regular publications of the Affiliated Groups include the following:

California Mathematics Council: *The California Mathematics Council Bulletin*—Dr. Edwin Eagle, San Diego State College, San Diego.

Colorado Council of Mathematics Teachers: *Bulletin of the Colorado Council of Mathematics Teachers*—Mr. Warren Geyer, 1133 State Street, Denver 6.

Mathematics Section Eastern Division Colorado Education Association: *Bulletin of the Mathematics Section Eastern Division Colorado Education Association*—Miss Ruth Hoffman, 338 Fox St., Denver.

Illinois Council of Teachers of Mathematics: *Newsletter of the Illinois Council of Tecahers of Mathematics*—Professor M. L. Hartung, University of Chicago.

Indiana Council of Teachers of Mathematics: *The Indiana Mathematics Teacher*—Miss Olive Leskow, 1806 W. 5th Ave., Gary.

Iowa Association of Mathematics Teachers: *Newsletter, Iowa Association of Mathematics Teachers*—Miss Viola Smith, Maquoketa High School, Maquoketa.

Kansas Association of Teachers of Mathematics: *Bulletin of the Kansas Association of Teachers of Mathematics*—Dr. Gilbert Ulmer, University of Kansas, Lawrence.

Minnesota Council of Teachers of Mathematics: *Minnesota Mathematics Newsletter*—Dr. Donovan A. Johnson, University High School, Minneapolis.

Nebraska Section of the National Council of Teachers of Mathematics: *Bulletin, Nebraska Section of the National Council of Tecahers of Mathematics*—Miss Mabel Nielson, Sidney.

The Association of Teachers of Mathematics in New England: *The ATMNE Newsletter*—Mr. Harland B. Garland, High School of Commerce, Boston.

Association of Mathematics Teachers of New Jersey: *The New Jersey Mathematics Teacher*—Miss Madeline D. Messner, Abraham Clark High School, Roselle.

Association of Teachers of Mathematics of New York City: *Bulletin of the Association of Teachers of Mathematics of New York City*—Mr. Abraham I. Goodman, New Utrecht High School, 1601 Scott Street, Brooklyn 14.

Ontario Association of Teachers of Mathematics and Physics: *Newsletter of the Ontario Association of Teachers of Mathematics and Physics*—Mr. H. E. Totten, Forest Hill C. I., Toronto.

Pennsylvania Council of Teachers of Mathematics: *Bulletin of the Pennsylvania Council of Teachers of Mathematics*—Dr. Catherine A. V. Lyons, 12 S. Fremont Ave., Pittsburgh 2.

Wisconsin Mathematics Council: *Wisconsin Teacher of Mathematics*—Dr. Ralph C. Huffer, Beloit College, Beloit.

HELP NEEDED IN SIX STATES

Mathematics teachers in thirty-nine states and the District of Columbia were associated with The National Council

(*Continued on page 248*)

Report of State Representatives

By M. H. Ahrendt
Chairman of State Representatives

At the end of the first semester of the present school year a questionnaire was sent to each of the state representatives of the National Council of Teachers of Mathematics. The 44 replies that have been received to date reveal some interesting data.

The primary concern of the representative is to spread the benefits of the work of the Council to as many persons as possible. This implies the urgency of maintaining and increasing membership. The representatives as a group appear to know that individual contacts are more persuasive than group contacts, for the number of individual contacts made by mail or in person under the direction of the 44 representatives is in excess of 6,600. Announcements about membership and other Council business have been made in 199 meetings. The total number of persons who heard these announcements exceeds 15,000. It should be remembered that these figures cover only half of the 1951–52 school year. Thus it is clear that during the full school year the total impact of the representatives is very great.

In addition the representatives have engaged in a large variety of other professional activities. A frequently mentioned activity was working with and trying to strengthen the local organizations of mathematics teachers. In several states where no group has yet formally affiliated with the Council, the representative is trying to help form such a group. Other typical activities have included the sending of Council publicity to local and state journals and the enlisting of the aid of several assistants in promoting the program of the Council.

The best results are achieved when the representative and the organized groups of mathematics teachers in a state work closely together. No representative likes to force his way into the activities of his state or local groups. He does not like to ask permission at meetings to make announcements for the Council or to solicit memberships. He should be invited to perform these services. If a representative is hesitant or inactive, he may even need a little urging. In those states where the representative and the organized groups have worked together, the Council has grown both in spirit and in numbers.

The responsibility of teachers of mathematics in the modern machine age is very great. The modern age could not have been built and cannot continue to exist without mathematics. Not only the scientists, engineers, and technicians need mathematics. Even the ordinary citizen who wants to live intelligently and appreciatively in the machine age must know some mathematics. Experience shows that mathematics teachers can discharge their responsibilities most effectively when they join together in professional groups such as the National Council of Teachers of Mathematics. Thus the work of the Council becomes of great significance. And one of the very important cogs in carrying on this work is the state representative.

Affiliated Groups

(*Continued from page 247*)

of Teachers of Mathematics through an Affiliated Group at the beginning of 1952. Plans for group affiliation in three of the other states are in progress. No teacher organizations or individual teachers are in correspondence with us about possible affiliation in the remaining states. These states are Idaho, Montana, Nevada, North Dakota, Washington, and Wyoming. If you live in one of these remaining areas, won't you help promote affiliation so that the Fourth Delegate Assembly in Atlantic City in 1953 may represent teachers in all of the forty-eight states.

Critical Thinking as an Aim in Mathematics Courses for General Education*

By James H. Zant
Oklahoma A. and M. College, Stillwater, Oklahoma

Introduction

Since this paper deals with critical thinking I want to be particularly careful to make the thinking represented by material discussed here also critical. Hence, the paper will begin with certain definitions or assumptions as to the meanings of such terms as General Education, Mathematics for General Education and Critical Thinking. It is too much to expect in a brief paper that concise definitions of such general terms are possible; it is less likely to expect unanimous agreement on the definitions that will be stated. However, to think clearly about such a topic as critical thinking as an aim in mathematics for general education it is necessary that the terms be defined.

It has been said that "general education had its impetus in a desire to get away from the routine of the conventional college pattern which too often tended toward a sort of dime-store arrangement where the student went to the counter that attracted him and bought more or less of this or that educational speciality."[1] General education has also been characterized, again indirectly, by the following statement: "General education as a movement has attempted to correct undue emphasis on specialization, on the acquisition of factual information and technical skills and the almost exclusive emphasis on the intellectual development of the student which has come to be associated with the more traditional educational programs. This movement is designed to encourage integration and retention of knowledge and skills gained by the students and to provide opportunity for emphasis on long-term goals of instruction." A more positive viewpoint is exhibited in the following statement: "Any attempt to give the answer by stating a prescribed curriculum and content is likely to prove futile, . . . general education itself is not a matter of content only. Rather it is an aim, a purpose, a philosophy, which may be realized in different ways on different stages in different communities. It is a matter of objectives, rather than subjects taught; a matter of a total effect to be gained, rather than the devices to be used to get the effect."[2]

This viewpoint has been expressed differently thus: "The primary end of all college-level instruction is the development of conceptual structures which will give the student intellectual control over the facts and phenomena of life. Under this view, the difference between general and specialized education becomes very clear. General education has its concepts oriented toward life and possibly built up through the ordinary experiences of the individual, while education for a specialist would involve those conceptual structures and intellectual methods necessary for the use of mathematics, for example, in a specialistic way."[3]

Hence in this paper general education will be thought of as a "philosophy" or a

* Read on August 22, 1950 at the Tenth Summer Meeting of the National Council of Teachers of Mathematics, Madison, Wisconsin.

[1] William N. Chambers, "Report of Conference Group 7," Fourth National Conference on Higher Education, Department of Higher Education of N.E.A., 1949.

[2] *Ibid.*

[3] Schiller Scroggs, in a personal letter dated June 28, 1950.

set of "conceptual structures" designed to enable the student to cope with those life situations which are outside of the mere vocational category. This is still not too definite, and defining and isolating these conceptual structures may take a considerable amount of time. Finding a curriculum that will eventuate in such changes in student behavior will probably take much longer. We are assuming, however, that the ability to do critical thinking in life situations is a desirable aim of general education.

Obviously if we accept this concept of general education, mathematics courses designed to contribute to these ends will be something different from the traditional courses in college or at least the aims will be different. Less emphasis will be placed on the learning of mathematical skills, theorems, etc., as such and more will be placed on fundamental meanings and concepts. A consideration of mathematics in such an educational scheme usually leads to some sort of general mathematics instead of the traditionally compartmentalized sequential courses. However, it is not possible to say that one scheme will lead to the desired changes in the student and that the other will not. Evaluation procedures and instruments which will tell which type of course is more effective in this sense have not been developed. In fact, it seems probable that there is no one solution to the problem of the subject matter or its organization for purposes of general education. It may well be that several widely different treatments will give the desired effects on the students.

Hence, in this paper mathematics for general education will mean that type of mathematics, any type of mathematics, which will further the aims of general education. These aims, to repeat, are "the development of conceptual structures which will give the student intellectual control over the facts and phenomena of life" outside the vocational category.

The Meaning of Critical Thinking

Critical thinking will be thought of as an ability to be acquired or improved by the student as a result of the various experiences he has had in school and out during a certain period. In general education we are concerned with effective living and the desired concepts should be oriented toward life and if possible be built up through ordinary life experiences. However, it is not necessary, if indeed possible, to draw all school experiences from life situations. It seems probable that the ability to do critical thinking in life situations may be improved by giving attention to the processes in ideal (mathematical) situations in addition to learning to attack real life situations from this point of view.

If a person is able to apply the abilities involved in critical thinking to a wide variety of problems which he meets in everyday living, we may say that he has the ability to do critical thinking in these areas. It is part of the task of general education to cause such changes in the student's knowledge, abilities and attitudes that he will do this type of thinking in his efforts to solve the various problems he will meet in life.

The fundamental concepts of mathematical thought as exhibited clearly in elementary Euclidean geometry furnish an excellent example of critical thinking. It has been said that it represents a perfect example of critical thinking. All mathematical reasoning is the same. As you know, it involves undefined terms, definitions, assumptions and deductively proved theorems. Though the assumptions of mathematics often seem to conform to experience, we know that such conformation is not necessary. In life situations we do not have this freedom of choosing or altering the assumptions and we must often base our argument and decisions on assumptions or evidence which may be doubtful at the time. In this sense we often "create" assumptions which we

may hope actually fit experience. Hence we see that mathematical thinking may be thought of as furnishing the ideal for all other critical thought.

No person familiar with modern thought in mathematical education will imply that mathematics teachers assume that ability to do critical thinking in mathematics will of itself transfer to other situations or problems. Such transfer must be sought and worked for by all the teachers in the school or college. Critical thinking is a conscious process; it involves careful steps such as definite statement or at least a careful consideration of the assumptions, a knowledge of the meaning and procedures of inductive reasoning, the ability to make logical inferences, the ability to recognize logical fallacies both in one's own work and in the work of others, the ability to know that the proof or solution of a problem is complete, and finally that the correctness of any solution is dependent on the assumptions and the validity of solutions of dependent problems or theorems.

To bring such a thought process to the level of consciousness in the student it is necessary usually to illustrate with examples that range from simple deductions based on well-known and universally accepted premises to more complex ones where the premises themselves may be in question. The beginnings of this task may well be the object of Mathematics for General Education. It is easy to state a few undefined terms, write a few definitions and a few assumptions and proceed immediately to deductive thinking in the proof of simple theorems. An example is the proof of the theorem that vertical angles are equal. This theorem is often proved as the first theorem in the high school course in plane geometry and is repeated here for illustrative purposes. The undefined terms, definitions and assumptions needed are first stated.

Undefined terms: *point, straight line, angle, straight angle, intersect, opposite, formed, portion, designated, side.*

Definitions:

1. When two straight lines intersect in a point the pairs of opposite angles formed are called *vertical angles*.
2. A portion of a straight line between two designated points is called a *straight line segment.*

Assumptions:

1. The sum of all angles on one side of a straight line about a given point is equal to a straight angle.
2. All straight angles are equal to each other.
3. Things equal to the same thing or equal things are equal to each other.
4. If equals are subtracted from equals, the results are equal.

Theorem. *If two straight lines intersect in a point, the vertical angles are equal.*

Given: Two straight line segments AB and CD intersecting at the point O. (See Figure 1.)

To prove: Angle BOD = angle AOC.

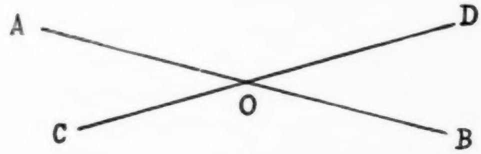

FIG. 1

Proof:

Statements	*Reasons*
1. Angle AOD + angle BOD = a straight angle	1. Assumption No. 1
2. Angle AOD + angle AOC = a straight angle	2. Assumption No. 1
3. Angle AOD + angle BOD = Angle AOD + angle AOC	3. Assumptions No. 2 and 3
4. Angle BOD = angle AOC	4. Assumption No. 4

From this point it is possible to go on to more and more complicated theorems which illustrate the various intricacies of logical thinking. These include the direct deductive type of proof illustrated in the theorem above, the indirect proof, the analysis which is often a basis preliminary to a deductive proof and finally mathematical induction which often enables us to prove statements arrived at inductively but not provable by standard deductive methods.

It is also possible and highly desirable for students interested in general education to examine mathematical systems whose assumptions are different and contradictory to those ordinarily accepted. One of the non-Euclidean geometries is often used for this purpose. The one proposed by Lobatchewsky, in which the assumption dealing with parallels is replaced by a contradictory one which says, in effect, that through a point not on a line two parallels may be drawn to the given line, is most often used. Naturally it leads to conclusions different in some respects from those of Euclidean geometry. This has the advantage of showing students that two or more systems of mathematics, each logically sound but based on different premises, lead to different (logically contradictory) conclusions.

These are examples of critical thinking in a rather limited field where it is not too hard to make changes in assumptions and definitions since emotions and attitudes of various sorts are not involved to the same extent or in the same manner as they are in the social or political areas of the student's life.

We might say that the responsibility of the mathematics teacher ends when he has taught his students the fundamentals of such critical thinking and when he has taught them to apply these skills in a wide variety of instances in the mathematical field. However, as stated above, these skills do not transfer automatically to other situations. Hence the mathematics teacher must assume his share of the responsibility for seeing that the student is able to apply this sort of thinking to other areas of human thought. Proposals for doing this at the secondary level have been made and have some merit for teachers of college level classes.

The Teaching of Critical Thinking

It has been said that every report on the teaching of mathematics which has appeared in the last fifty years has stated that the development of critical thinking is an aim of the teaching of mathematics.[4] Certainly many of them have included among the objectives recommended such things as "clear thinking," "problem solving," "scientific thinking," "nature of proof" and the like. Such statements may be found in the *Fifteenth Yearbook*[5] and the report *Mathematics in General Education*.[6]

It seems therefore to be a consensus of opinion that important values to be derived from the study of mathematics, particularly demonstrative geometry, are an aquaintance with the nature of proof and a familiarity with postulational thinking as a method of thought. These skills then will be available for use not only in the study of mathematics but in other fields of thought as well.[7]

"The assumption which mathematics teachers are making is that since demonstrative geometry offers possibilities for

[4] M. F. Rosskopf, The Mathematics Teacher, XLIII (April 1950), 142.

[5] *The Place of Mathematics in Secondary Education*, report of the Joint Commission of the Mathematical Association of America and The National Council of Teachers of Mathematics, pp. 39–40. Fifteenth Yearbook of the National Council of Teachers of Mathematics, New York: Bureau of Publication, Teachers College, Columbia University, 1940.

[6] *Mathematics in General Education*, Report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum, p. 59, New York: D. Appleton-Century Co., 1940.

[7] H. P. Fawcett, *The Nature of Proof*, Thirteenth Yearbook of the National Council of Teachers of Mathematics, p. 6. New York: Bureau of Publications, Teachers College, Columbia University, 1938.

development of critical thinking, this sort of thinking is *necessarily* achieved through a study of the subject. Such an assumption has not been validated and the results of past experience indicate that it should be seriously questioned. To theorize concerning values which are believed to be the unique contribution of demonstrative geometry to the general education of young people is not a difficult matter, but to plan and carry out this program in such a way that these desired outcomes are actually realized is a problem which has not been squarely faced by teachers of mathematics."[8]

These statements indicate that though mathematics teachers of various levels were agreed that geometry at the high school level is important in training students to do critical thinking in all areas, there were some doubters even among those most interested in the teaching of geometry. Among these were Dr. Fawcett, quoted above, who set out to study the problem carefully and get some definite answers as to what should be done at this level.

He made assumptions to the effect that students have already done critical thinking before beginning the study of geometry, that they should be allowed to reason about geometry in their own way by the logical processes of tenth grade students rather than those of textbook writers or teachers and that opportunities should be provided for the application of the postulational method to non-mathematical material. Dr. Fawcett allowed classes to reason about geometry in their own way, depending on their environment, realizing that the teacher who is acquainted with the domain which the pupil is about to enter, is definitely a part of the environment. The particular sequence of theorems to be covered was not a matter of concern, "since the emphasis is to be placed on the nature of the process by which these theorems are proved and not on the theorems themselves."

This process yielded a geometric sequence quite different from our conventional geometry but it was based on a number of undefined terms, definitions, assumptions or postulates accepted by the students and included a number of propositions proved by *all* students as well as a number which were proved by only a part of the group.[9]

The class discussion of this particular experiment began with an exploration of the meaning of words drawn from the student's experience. These early discussions, which lasted for about a month, often involved personal opinions and prejudices, as in the case of attempts to define "aristocrat" and "labor class." Later, after practice in thinking in the field of geometry where emotional biases were not present, the students were led back to these non-mathematical situations and the desirability of thinking about such matters in an objective fashion was stressed.

Situations drawn from the public life of the state were used to show the importance of definition, as a case of the Restaurant Law in Ohio, in which it became necessary for the state to set up an official definition of a restaurant. Other public questions were submitted by the students and discussed in class.

Similar procedures were used in other phases of the fundamentals of critical thinking, that is, they were discussed from the standpoint of geometry and then the class passed to actual situations in public or private life. For example, they studied the difference between forming a conclusion based on a few special cases and actually finding a proof of the theorem or statement. The problem of the Soldier's Bonus was before the Congress in 1936. Assumptions were often made after the bill had finally been passed over the President's veto, that senators who voted

[8] *Ibid.*, p. 10.

[9] *Ibid.*, Chapter IV.

against the bill would not be reelected. These assumptions or opinions came from a limited number of special cases and the students were able to point out the fallacies in various arguments.

Advertisements which made numerous tacit assumptions to "prove" their points were submitted and studied by the class. So also were petitions for injunctions, editorials and the like. The class was asked to analyze these illustrations of public thinking to find which statements were assumptions and which were proved.

The students also came to realize that underlying many of their beliefs were assumptions that would bear scrutiny. Out of this came a request that the examination of these assumptions become a matter of group consideration. From this activity came a critical analysis of such topics as racial superiority, compulsory education, a citizen's obligation to his government in time of war, and the like.

Dr. Fawcett made the following generalizations about his study:

"1. Mathematical method illustrated by a small number of theorems yields a control of the subject matter of geometry at least equal to that obtained from the usual formal course.

"2. By following the procedure outlined—it is possible to improve the reflective thinking of secondary school pupils.

"3. This improvement of the pupil's ability for reflective thinking is general in character and transfers to a variety of situations.

"4. The usual formal course in demonstrative geometry does not improve the reflective thinking of the pupils."[10]

Dr. Fawcett further stated " . . . that the study of proof should not be considered as a course which a pupil begins at a certain point in his secondary school experience and which he completes at the end of a given time. To encourage a pupil to think that he understands all there is to know about proof because he had a course on that topic is to ignore the fact that even the most respected mathematicians disagree on what a proof is. There are, however, aspects of this important topic which the pupils in our secondary schools can understand and which in the opinion of the writer, contribute effectively to the general education of these young people. The concept of proof is one concerning which the pupils should have a growing and increasing understanding. It is a concept which not only pervades his work in mathematics but is also involved in all situations where conclusions are to be reached and decisions to be made. Mathematics has a unique contribution to make in the development of this concept, and up to the present time teachers of mathematics have, in general, assumed that this contribution can best be made in the tenth year through the study of demonstrative geometry. The practice resulting from this assumption has tended to isolate the concept of proof, whereas this concept may well serve to unify the mathematical experiences of the pupil."[11]

Considering the discussion above we may well say that teaching the students the meaning and use of critical thinking, not only in mathematical situations but in enough life situations so that they will understand clearly the use of this knowledge and skill in many fields, is perhaps the chief aim of mathematics for general education. This is not saying, of course, that people cannot learn to think clearly and critically without the study of mathematics. It does mean, however, that those who think clearly use many of the same fundamental methods and skills as those employed in mathematics. If the study of mathematics with the viewpoint outlined here can help a person acquire and use these skills of critical thought, then it will undoubtedly

[10] *Ibid.*, p. 119.

[11] *Ibid.*, pp. 119–20

prove valuable as a part of the curriculum of general education.

Evaluation of the Ability to Do Critical Thinking

The method of evaluation arises immediately as one of the vital problems connected with the teaching of critical thinking. How will we measure, in terms of the student behavior, the change that has taken place in the individual? Many college teachers who have given thought to the complete problem of critical thinking are reluctant to classify it as a single skill. Rather they believe that a battery of skills are involved. This suggests that a battery of tests may be required. An examination of some of the tests in current use, like the General Educational Development Test, The American Council on Education Psychological Examination and the like, reveals that certain parts of them seem to test some of the abilities involved in critical thinking or some parts of the ability. These tests either do not indicate what parts of the complete skill are being tested or they obviously deal with certain subsidiary skills and exclude others. Another criticism indicates that language ability and/or general knowledge may be more important in deciding what answers should be given to the test item than the ability to do critical thinking. All of this indicates that much work must still be done on the selection of test items even assuming that there is agreement on the meaning of critical thinking itself.

Since our aim is to train the student so that he can and will do critical thinking when faced with life problems we may list the following criteria for the selection of situations or problems to be used in an evaluation program.

1. The situations should be realistic in terms of student experience—not hypothetical but recognizable as actual in their experience or the experience of people known to them. Since to think critically a person must learn to ignore irrelevant material, the possibility of involvement of emotional reaction or of attitudes should not in itself be the basis of elimination of a situation.

2. The situations should be of such non-provincial nature as to be usable by institutions in various parts of the country.

3. The situations should be so selected that the evaluation device may be used from time to time without being invalidated through emphasis on matters of transitory importance.

4. The situations should be independent of particular courses and content so as to avoid the overlapping with *special area projects* and to make some provision for evaluating the total growth of critical thinking from all sources.

5. The situations should be so set up as present all pertinent information or be so qualified that any information not so presented may be expected to have been attained by most students. The vocabulary should likewise be within the range of the majority of students.

6. Directions should be clear and less complicated than the thinking processes tested.

While it is plain from the criteria just listed that an attempt is being made to eliminate test items wholly dependent on mathematical knowledge and skills, as well as those dependent on any other *special areas,* there is no reason why test items based on the above situations should not be used in a class in mathematics for General Education since one of our primary purposes is to teach critical thinking in general situations.

The building of such a test will be a difficult job. As indicated above the existing tests which include parts which may be assumed to deal with critical thinking have a wide variety of approaches. However, it is probable that something definite will be done about it soon. The Cooperative Study of Evaluation in General Education of the American Council on Education is including work on such a test in its program.

Evaluation of Courses in Mathematics for General Education

A final problem involved in the discussion of mathematics for general education is an evaluation of the various courses being taught in American colleges for that purpose. Some of the questions which arise are: Is there any one best course? and are there different kinds of courses which seem to accomplish the same ends, even though their organization and content may vary widely?

To answer such questions as these we must

1. Have a clear understanding of the objectives of our courses and their meaning in terms of student behavior.
2. Develop, if we have not already done so, devices or techniques of collecting evidence on such student behavior.
3. Collect such evidence at two or more stages of development so that results may be interpreted in terms of changes in the student.
4. Compare the gains of similar groups of students exposed to different types of educational experience or courses.

Such a scheme involves some agreement on objectives but it does not involve absolute agreement. We have assumed agreement on the ability to do critical thinking. There would probably be enough other common objectives to use as a basis of comparison of courses. Since the objective of the ability to do critical thinking is of such a complex nature, courses could be compared on that basis alone. The big problem, as stated above, is in developing a satisfactory measuring instrument.

On the basis of the brief definitions of general education, mathematics for general education, and critical thinking, it seems reasonable and necessary to consider the ability to do critical thinking not only in mathematics but also in general life situations as a desirable aim of courses in mathematics for general education. This poses other problems of constructing or finding tests which will show whether this aim is being accomplished and also setting up some methods of evaluating various courses which may be proposed to accomplish this aim.

HAVE YOU SEEN?

In *The Australian Mathematics Teacher* for November 1951
- "Functional Arithmetic" by J. G. Nay
- "Some Elementary Number Theory" by J. P. McCarthy
- "A Speed Multiplication Test" by G. Rush

In *The American Mathematical Monthly* for December 1951
- "Note on the Law of Cosines" by S. L. Thompson

In *American Journal of Physics* for January 1952
- "Units and Dimensions in Physics" by C. H. Page
- "Mathematics in the Undergraduate Curriculum" by Joseph L. Rood

In *The Mathematical Gazette*, for December 1951
- "The Mathematics of Easter" by E. J. F. Primrose
- "Trigonometry in the Main School"
- "Mathematical Notes: A Symmetrical Figure to Demonstrate Pythagoras' Theorem; Approximation of $\sqrt{x}$"

In *School Life* for February 1952
- "Engineering and the High School as a Source of Supply" by Harry A. Jager and Henry H. Armsby

In *Life Magazine* for February 18, 1952
- "Wizard of 0000s." Antoon van den Hurk of Helenaveen, Holland mentally multiplies 6241082426 by 38254319074 in 21 minutes.

In *Scientific American* for March 1952
- "Logic Machines" by Martin Gardner describes the progress which has been made on a device for testing the validity of a system of thought.
- "The Amateur Astronomer" by Albert G. Ingalls describes a method of finding your latitude without the use of a precision instrument.
- "James Clerk Maxwell's Poetry" by I. Bernard Cohen
- "The Quantum Theory" by Karl K. Darrow
- "Fellowships Taxable?"

Some Notes on the Prismoidal Formula

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Introduction

In this article we shall discuss the wide applicability of the prismoidal formula to the calculation of the volumes of many common solids, as well as many that are not so common. Some explicit criteria for the application of the formula will be presented. The writers believe that the prismoidal formula may serve as an excellent review of most of the volume relationships of solid geometry, as well as a review of all the geometric relationships necessary to obtain the area of the bases and midsections.

A *prismoid* is a special case of a prismatoid. A *prismatoid* is a polyhedron (a solid bounded by planes) all of whose vertices lie in two parallel planes. The faces of the polyhedron lying in the two parallel planes are called the *bases* of the prismatoid. If the two bases of the prismatoid have the same number of sides, the prismatoid is called a *prismoid*. [2]* If those edges of a prismoid not lying in the bases of the prismoid are parallel, the solid is called a *prism*. The prismoidal formula may be used for many solids including all prismatoids and therefore for all prismoids and prisms. Some high school textbooks only consider the application of the prismoidal formula to prismatoids. Other textbooks have exercises that demonstrate the use of the formula in finding volumes of more general solids. In particular, Frame's *Solid Geometry* [2] contains an excellent treatment of the subject matter of

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The volumes of a

Rectangular parallelepiped
Cylinder
Prism
Pyramid
Cone
Prismatoid
Prismoid
Sphere
Spherical segment
Spheroid
Conoid
Wedge
Ellipsoid, and many other solids.
(See Section B)

$$=\frac{H}{6}(B_1+4M+B_2)$$

$H=$ Altitude
$B_1=$ Area of first base
$B_2=$ Area of second base
$M=$ Area of midsection

The *prismoidal formula*, $V=H/6(B_1+4M+B_2)$, is considered in many high school solid geometry textbooks in connection with a topic on the prismatoid. In these texts the formula is frequently called the prismatoid formula; however, in several calculus textbooks and books of tables this formula is called the prismoidal formula. The writers shall use the latter term in discussing the formula throughout this paper.

* Numbers in brackets refer to references listed at the end of the paper.

this paper. This formula is applicable not only to most of the solids usually studied in a high school solid geometry course, but also to many solids having unusual shapes. For example, the prismoidal formula can be used to find the volumes of many of the solids appearing in the exercises of calculus textbooks.

When can the formula be used? When can it not be used? In an attempt to answer these questions, the remainder of the paper is divided into two sections. The first section (Section A) contains examples of the use of the prismoidal formula for most of the common solids, and a few solids that are frequently considered in calculus textbooks. The second section (Section B) contains some explicit criteria for the general applicability of the formula.

A. Application of the Prismoidal Formula to Common Solids

I. Solids for Which $V = BH$

We shall consider here solids that have two parallel bases. Any section of the solid by a plane parallel to these bases is called a *principal section*. If all the principal sections of a solid have equal areas, the volume of the solid is equal to the area of its base multiplied by its altitude. Such a solid might be illustrated by a uniform pack of cards stacked on a table in any manner which leaves the cards parallel to the table top. The common solids for which $V = BH$, however, can generally be described as being enclosed by a closed cylindrical or prismatic surface and two parallel planes that intersect all elements of this surface. These solids are called cylinders and prisms. For each of these, the application of the prismoidal formula is almost trivial since the two bases and the midsection are congruent, i.e., $B_1 = M = B_2$ and thus the prismoidal formula becomes

$$V = \frac{H}{6}(B_1 + 4M + B_2) = \frac{H}{6}(6B_1) = BH.$$

II. Solids for Which $V = BH/3$

These solids include right pyramids, right circular cones, and in general, all solids enclosed by a pyramidal or conical surface and a plane cutting all the elements of the surface. The prismoidal formula can be easily applied to these solids because the area of one base is zero (we shall take $B_2 = 0$) and the area of the midsection is always one-fourth of the area of the other base, i.e., $M = B_1/4$. For example, any circular cone may be considered as having $B_1 = \pi r^2$, $B_2 = 0$, $M = \pi(r/2)^2 = \pi r^2/4$, and height H.

$$V = \frac{H}{6}\left(\pi r^2 + \frac{4\pi r^2}{4} + 0\right) = \frac{\pi r^2 H}{3} = \frac{BH}{3}.$$

III. Solids for Which $V = BH/2$.

We shall briefly mention two types of solids, wedges and conoids, for which $V = BH/2$. A *wedge* may be defined as a triangular right prism with one face that is not a base of the prism taken as a base of the wedge. All principal sections are then rectangles such as $HIJK$ in Figure 1.

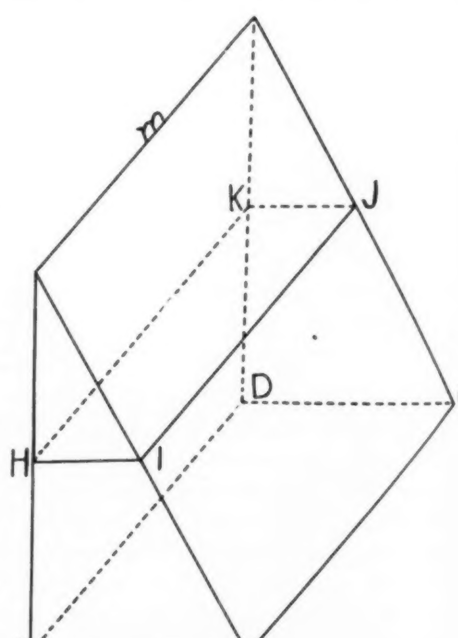

FIG. 1

A wedge may also be defined as a solid having a rectangular base $ABCD$; a line m parallel to a side of the rectangle and not in the plane of the rectangle; and its remaining faces determined by the totality of lines perpendicular to m and joining points of m to points of the rectangle. This definition may be readily modified to obtain the definition of a conoid. A *conoid* may be considered as a generalized wedge. Specifically, given any simple closed plane curve C and a line m not in the plane of C, but parallel to that plane, the totality of lines that are perpendicular to m and join points of m to the curve C together with the plane of C bound a conoid (Figure 2). In each of the above

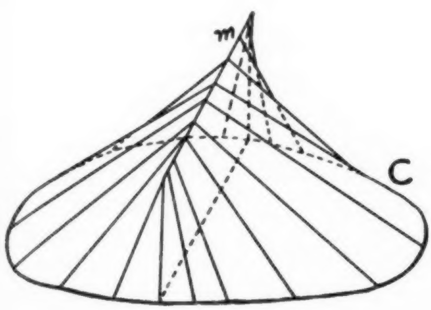

FIG. 2

types of solids, the area of the midsection may be taken as one-half the area of the first base and the second base may be taken as zero. Thus for all wedges and conoids

$$V=\frac{H}{6}\,(B_1+4B_1/2+0)=B_1H/2=BH/2.$$

IV. Volumes of a Sphere and a Spherical Segment

In the case of a sphere, we may take the first base at the south pole ($B_1=0$), the second base at the north pole ($B_2=0$), and the area of the midsection, i.e., the area enclosed by the equatorial circle as $M=\pi a^2$, and the height $H=2a$. The prismoidal formula then gives

$$V=\frac{2a}{6}\,(0+4\pi a^2+0)=4\pi a^3/3.$$

The prismoidal formula is also applicable to finding the volume of a spherical segment. It can be demonstrated that the radius r_m of the midsection is dependent upon the radius of the first base r_1, the radius of the second base r_2, and the altitude of the segment H. This dependence is expressed by the relationship

$$r_m{}^2=r_1{}^2/2+r_2{}^2/2+H^2/4.$$

An application of the prismoidal formula now gives

$$V=\frac{H}{6}\left[\pi r_1{}^2+4\pi(r_1{}^2/2+r_2{}^2/2+H^2/4)+\pi r_2{}^2\right]$$

from which we may obtain the usual formula for the volume of a spherical segment

$$V=\frac{H}{6}\,\pi(3r_1{}^2+3r_2{}^2+H^2).$$

V. Volumes of a Frustum of a Cone and a Frustum of a Pyramid

The prismoidal formula is also applicable to finding the volumes of a frustum of a cone and a frustum of a pyramid. It is not difficult to show that the area of the midsection (M) of a pyramid is related to the area (B_1) of the first base and (B_2) of the second base as indicated by the relationship

$$M=(\sqrt{B_1}+\sqrt{B_2})^2/4.$$

An application of the prismoidal formula now gives

$$V=\frac{H}{6}\,[B_1+4(\sqrt{B_1}+\sqrt{B_2})^2/4+B_2]$$

from which we may obtain the usual formula for the volume of a frustum of a pyramid

$$V=\frac{H}{3}\,(B_1+B_2+\sqrt{B_1B_2}).$$

In a similar manner the area of the midsection of a frustum of a cone is related to the areas and therefore the

radii of the two bases as indicated by the relationship

$$M = \frac{\pi}{4}\,(r_1^2 + 2r_1r_2 + r_2^2).$$

In this case, the application of the prismoidal formula again results in the usual formula for the volume of a frustum of a circular cone

$$V = \frac{\pi H}{3}\,(r_1^2 + r_1r_2 + r_2^2).$$

VI. Volumes of Ellipsoids and Spheroids

The prismoidal formula is also applicable to the ellipsoids. Consider the ellipsoid

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1.$$

As in the case of the sphere we may take $B_1 = B_2 = 0$ where the bases are in the planes $z = c$ and $z = -c$. The midsection of the ellipsoid then lies in the xy plane and is bounded by the ellipse $x^2/a^2 + y^2/b^2 = 1$, $z = 0$, which has area πab. Thus the prismoidal formula gives the usual result

$$V = \frac{2c}{6}\,(0 + 4\pi ab + 0) = 4\pi abc/3.$$

The volume of the oblate spheroid, the prolate spheroid, and the sphere can be considered as special cases of the volume of the ellipsoid for suitable values of a, b, and c. These three cases may be taken respectively as

$$V = 4\pi a^2c/3 \quad \text{where} \quad a = b > c,$$
$$V = 4\pi a^2c/3 \quad \text{where} \quad a = b < c, \quad \text{and}$$
$$V = 4\pi a^3/3 \quad \text{where} \quad a = b = c.$$

VII. Volume Common to Two Intersecting Right Circular Cylinders

Many calculus textbooks give the following problem: The axes of two right circular cylinders of radius a intersect at right angles. What is the volume common to the two cylinders? This problem can be easily solved by use of the prismoidal formula. Consider one-eighth of the total desired volume, for example, $ABCDE$ in Figure 3 where ED and EC are radii of one cylinder, and ED and EA are radii of the other cylinder. Let us take the first

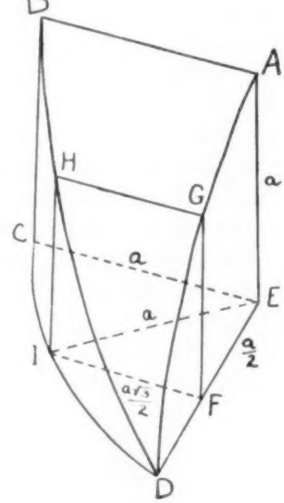

FIG. 3

base as the area of $ABCE$, the midsection as the area of $GHIF$, and the second base at D, $B_2 = 0$. We next use the fact that the faces EAD, ECD, and ECA are in mutually perpendicular planes, and the plane of $GHIF$ is parallel to that of $ABCE$. In the face ECD, we consider the right triangle EFI where $EI = a$, $EF = a/2$, and by the Pythagorean theorem $FI = a\sqrt{3}/2$. Similarly $GF = a\sqrt{3}/2$. It is then easy to see that $GHIF$ is a square of area $M = 3a^2/4$. Thus we have

$$V = \frac{a}{6}\,[a^2 + 4(3a^2/4) + 0] = 2a^3/3$$

as the volume of $ABCDE$. The desired volume is then

$$8(2a^3/3) = 16a^3/3.$$

The prismoidal formula may also be applied directly to the total volume common to two intersecting right circular cylinders [1]. In this case the cross-sectional areas are then squares with sides double those of the corresponding squares considered under the above method. Thus $H = 2a$, $B_1 = 0 = B_2$, $N = 4a^2$, and

$$V = \frac{2a}{6}(0+16a^2+0) = 16a^3/3.$$

VIII. Two Other Problems

Calculus textbooks often contain a problem similar to the following: In cutting down a tree 6 ft. in diameter, a cut is first made horizontally half way through the tree. A second cut is inclined at an angle of 45° to the horizontal and meets the first cut along a diameter of the tree. Compute the volume of the cylindrical wedge cut out (Figure 4).

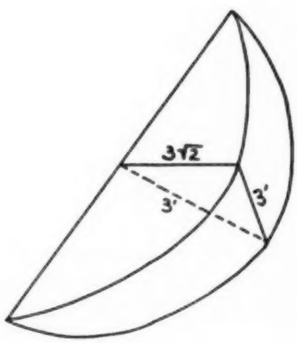

Fig. 4

The midsection of this solid is an isosceles right triangle with the legs 3 ft. Then $M=\frac{1}{2}\cdot 3\cdot 3=9/2$ sq. ft., $B_1=0$, $B_2=0$, $H=6$ ft., and the prismoidal formula gives

$$V = \frac{6}{6}[0+4(9/2)+0] = 18 \text{ cu. ft.}$$

Another typical calculus problem involves the calculation of the volume in the first octant that is interior to the cylinder having the line $x=-z$, $y=0$ as axis and intersecting the xy plane in the circle $x^2+y^2=a^2$ (Figure 5). This volume may be obtained using the prismoidal formula in which the first base is taken as the triangle in the xz plane, $B_1=a^2/2$, the midsection is an isosceles right triangle, $M=3a^2/8$; $B_2=0$; and $H=a$. Thus we have

$$V = \frac{a}{6}[a^2/2+4(3a^2/8)+0] = a^3/3.$$

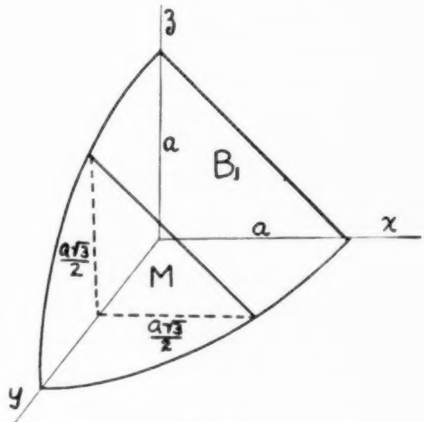

Fig. 5

Many other applications of the prismoidal formula could be given, however, we now turn to the task of determining whether or not the formula may be used for a given solid.

B. General Applicability of the Prismoidal Formula

In Section A several special cases of the application of the prismoidal formula were considered. The question still remains concerning the conditions for general application of the formula. Under what conditions can it be applied? Under what conditions can the formula not be applied? We now show that the prismoidal formula in general may be used for all solids having parallel bases and such that the area of every principal section is a polynomial of degree at most three in the distance of the section from a plane parallel to the bases. In other words, given a solid with parallel bases, we shall consider a coordinate system as in Figure 6 such that the area of any principal

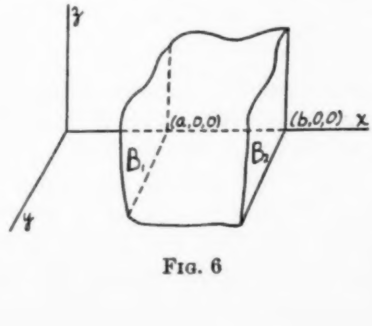

Fig. 6

section is a polynomial in the distance x of that section from a fixed plane parallel to it, and show that the prismoidal formula holds whenever $f(x)$ is a polynomial of degree not greater than three. The proofs are based upon the methods of the integral calculus.

Given a solid as described above, we have $B_1=f(a)$, $B_2=f(b)$, $M=f[(a+b)/2]$. Since $f(x)$ is a polynomial, it is integrable and the volume of the solid may be expressed in the form

$$V=\int_a^b f(x)dx.$$

Whenever $f(x)$ is a polynomial of degree at most three, i.e., $f(x)=Ax^3+Bx^2+Cx+D$, the above definite integral may be evaluated as follows to obtain the prismoidal formula.

$$\begin{aligned}V&=\int_a^b (Ax^3+Bx^2+Cx+D)dx\\&=\left.\frac{Ax^4}{4}+\frac{Bx^3}{3}+\frac{Cx^2}{2}+Dx\right]_a^b\\&=\frac{A}{4}(b^4-a^4)+\frac{B}{3}(b^3-a^3)\\&\quad+\frac{C}{2}(b^2-a^2)+D(b-a)\\&=(b-a)\left[\frac{A}{4}(b+a)(b^2+a^2)\right.\\&\quad\left.+\frac{B}{3}(b^2+ba+a^2)+\frac{C}{2}(b+a)+D\right]\\&=\frac{(b-a)}{6}\left[f(a)+f(b)+4f\left(\frac{a+b}{2}\right)\right]\\&=\frac{(b-a)}{6}[B_1+B_2+4M].\end{aligned}$$

Thus the prismoidal formula holds whenever $f(x)=Ax^3+Bx^2+Cx+D$ for all x satisfying $a\leqq x\leqq b$. The coefficients A, B, C, D may be any real constants positive, negative, or zero.

If $f(x)$, $g(x)$, and $h(x)$ are any polynomials such that $f(x)=g(x)+h(x)$ then $\int f(x)dx=\int g(x)dx+\int h(x)dx$. Thus if $f_4(x)=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4$, we may consider $f_4(x)=f_3(x)+a_4x^4$. Since the prismoidal formula may be used whenever the areas of the principal sections may be represented by $f_3(x)$ it may be used for $f_4(x)$ if and only if it may be used for a_4x^4. In general, if the prismoidal formula may be used for $f(x)$, it may be used for $f(x)+cx^k$ if and only if it may be used for x^k.

If $f(x)=x^n$ we shall consider the cases $n=0$ and $n>0$. If $n=0$ then $f(x)$ is 1, the volume is $b-a$, and the prismoidal formula applies. If $f(x)=x^n$ and $n>0$, then by integration $V=(b^{n+1}-a^{n+1})/(n+1)$ whereas by the prismoidal formu'a we have

$$V=\frac{(b-a)}{6}\left[a^n+4\left(\frac{b+a}{2}\right)^n+b^n\right].$$

These two values cannot be identically equal for all values of a and b unless the two coefficients of b^{n+1} are equal, i.e.,

$$\frac{1}{n+1}=\frac{1}{6}\left[1+\frac{1}{2^{n-2}}\right]\quad\text{or}\quad 2^{n-2}=\frac{n+1}{5-n}\cdot$$

Since this condition is satisfied only for $n=1$, 2, or 3, the prismoidal formula does not hold for $n>3$. In general, when $f(x)$ is a polynomial, the prismoidal formula holds if and only if $f(x)$ has the form Ax^3+Bx^2+Cx+D. This condition is satisfied for many common solids (Section A). It may fail to apply in general when $f(x)$ is a polynomial of degree greater than three or when $f(x)$ is not a polynomial, as in the case of a double cone. The above method of proof also applies whenever $f(x)$ has a Taylor series expansion. Thus the prismoidal formula does not in general apply if $f(x)$ is one of the functions having a Taylor series expansion that contains a term of degree greater than three. The trigonometric, logarithmic, and exponential functions are of this type.

It is a good exercise to verify that $f(x)$ has the proper form in each of the examples considered in Section A. For example, in the problem of two intersecting cylinders

(VII, Section A), all of the principal sections parallel to the base $ABCE$ are squares. If, in Figure 3, x is the distance of the principal section from the base $ABCE$, then one side of the square is $\sqrt{a^2-x^2}$. The area of a principal section is then (a^2-x^2) and is a polynomial in x of the form Ax^3+Bx^2+Cx+D where $A=C=0$, $B=-1$, and $D=a^2$. Thus the prismoidal formula is applicable to this particular solid.

The importance of these criteria for the use of the formula is illustrated by the following example. Given the parabola $y^2=4-x$, consider the volume bounded by the plane $x=0$ and the surface generated by revolving the parabola about the x-axis. We may take the base $B_1=4\pi$ in the plane $x=0$, $B_2=0$, and in general, $f(x)=\pi y^2=\pi(4-x)$. Thus $f(x)$ is a linear polynomial, and the prismoidal formula may be applied to give

$$V=\frac{4}{6}\left[4\pi+4(2\pi)+0\right]=8\pi.$$

Now consider the solid bounded by revolving the parabola $y^2=4-x$ about the y-axis. In this case we take the midsection in the plane $y=0$, and all principal sections are circles. The area of each principal section has the form

$$f(y)=\pi x^2=\pi(4-y^2)^2,$$

and the prismoidal formula does not apply since $f(y)$ is of the fourth degree.

A more complicated formula applicable when $f(y)$ is of degree at most five may be found in [3].

We have seen how the prismoidal formula is applicable to a large number of common solids, as well as to many that are not so common. Some explicit criteria for the application of the formula have been presented. Many students enjoy learning of broad generalizations that are applicable to a great number of special cases. Here then is an opportunity to teach for enjoyment as well as for practical application.

References

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2. Frame, James S. *Solid Geometry*, New York: McGraw-Hill Book Co., Inc., 1948, 95–138.
3. ———. "Numerical Integration," *The American Mathematical Monthly*, L (April 1943), 244–50.
4. Swenson, John A. and Bakst, Aaron. *Notes on the Professionalized Matter in Senior High School Mathematics*. Part II, 22–25. New York, 1931.

Chicago's Museum of Science and Industry announces a new exhibit on geometry which should be of interest not only to teachers and students in the Chicago area but to those who will be visiting there in the forthcoming months. The exhibit presents geometry as being "amazing, ingenious, beautiful and profound" and was designed by Dr. Karl Menger, professor of mathematics at the Illinois Institute of Technology. A pamphlet describing some of the material shown in the exhibit entitled "You Will Like Geometry" may be purchased at the Museum for 10 cents or will be mailed from the Book Mart of the Museum if the order is accompanied by 13 cents in stamps or coin.

That geometry is ingenious is illustrated by the solution of a problem formulated by the Emperor Napoleon: How to construct the corners of a square without the use of a straightedge. The best approximate trisection of angles discovered by a German master tailor in 1930, and a good approximate method for squaring the circle by the Chicagoan Johnston in 1950 are also displayed.

A simple instrument demonstrates how a cycloid may be drawn and how this curve is related to the famous squaring the circle problem. Some of the properties and uses of this curve are shown. For example, in the Museum exhibit are three large transparent pipes: one has the shape of a cycloid; one looks like a quarter of a circle; and one is straight. All three pipes begin at the same point and end at the same point. Pressing a button drops three balls simultaneously, one into each of the pipes. Which ball will reach the bottom first is the question. Some people guess that it will be the ball in the straight pipe because it travels the shortest path. But the path in the circular pipe acquires a greater speed in the early part of the motion and reaches the bottom earlier. However, the ball which reaches the bottom first is the one which travels in the longest of the three pipes although it dips below the end level and then rises again. The path of fastest descent is a cycloid.

Some results of the most modern geometric studies are also on display, among them objects which have probably never been exhibited before. One wall of the exhibition is devoted to very simple but important concepts of modern geometry, such as symmetry and duality which have helped physicists clarify their ideas about the arrangement of molecules in crystals and about the structure of atoms. Another section of the exhibit shows the various star-shaped and composite regular polyhedrons.

The Use of a Ruler in Teaching Place Value in Numbers

By J. T. Johnson
Chicago Teachers College, Chicago, Illinois

This article will attempt to show how a very simple visual aid, available in every school room, was utilized to great advantage in teaching the meaning of place value in arithmetic. This is a phase of structural meaning* and is fundamental as it underlies the very basis upon which our number system is built. It is, furthermore, intrinsically related to operational meaning* and should, therefore, be among the first of the meanings to be taught.

Some of us seem to think that the teaching of meanings will solve most of our teaching problems in arithmetic, forgetful of the fact that the understanding of these meanings by the pupils must also be assured. These understandings require a certain degree of maturity of mind and concept mastery that younger children do not always possess. Our human race struggled with the concept of place value for hundreds of years before it was accepted for use. Many of us take it for granted, however, in children of the third and fourth grades.

The sixth and seventh grades were chosen in which to present this work because here the pupil has had or should have had an abundance of exercises in reading and writing of whole numbers and some teaching of the meaning of place value of the different figures of a whole number. Now he is learning decimals and should be ready for an extension of the concept of place value in decimals.

In hunting for an effective visual aid to give objective evidence that the place value of a figure in any number, whole or decimal, is ten times the place value of the next figure to the right or one tenth of the place value of the next figure to the left the common school ruler was selected because it has centimeter divisions on one side and is often available, as in Chicago, as regular classroom equipment. Students often prefer to buy their own individual rulers like those which come in plastics of various colors. They can be had for a dime in any five and ten cent store and are well graduated with the centimeter and inch marks on opposite edges, both of which are usually beveled.

It cannot be taken for granted that the sixth grade student has sufficient acquaintance with the centimeter markings on his ruler. Therefore a preparatory exercise in measuring lengths and widths to the nearest inch and to the nearest centimeter of various sized cards and papers was given. In this exercise questions were asked by pupils if they could make a closer measure than the nearest centimeter especially on the small cards. This led naturally to the study of the small millimeter and its relation to the centimeter. In this study the pupil could see and count the actual number of millimeters in a centimeter. They discovered that one centimeter and ten millimeters are one and the same thing, that is they are identical. An aid often used in the earlier grades in teaching place value involves the use of coins such as dimes and pennies. We can lay out ten pennies and one dime side by side for the pupil and he can be made to see that they are equal in value but we cannot squeeze the ten pennies into the space of one dime as is done with the ten millimeters occupying the same identical space as the one centimeter. The one dime and ten pennies are equal in

* There are at least three quite distinct phases of meaning in arithmetic. See writer's article, "What Do We Mean by Meaning in Arithmetic," The Mathematics Teacher, XLI (December 1948), 362–67.

value but the ruler goes one step better and as a visual aid shows that the centimeter and the ten millimeters are identical.

The class was next given exercises in finding different lengths in millimeters on their rulers such as 12 mm., 57 mm., 105 mm., 215 mm., 295 mm., 304 mm., etc., then such points as 2 cm., 2 mm.; 23 cm., 8 mm.; 30 cm., 4 mm.; etc. The teacher can check easily on this as a class exercise by asking pupils to place their thumbs on their rulers with nails on the desired mark called for and then hold rulers high up over heads showing thumbs on rulers in plain view for the teacher as he moves around the room.

In the next exercise the pupils were asked to measure to the nearest millimeter as had been their desire before. Some clean cut cards should be available for this. Some old unused tickets will serve in the absence of library cards. They were asked to measure first the length and width of the cards to the nearest centimeter, then to try to measure to the more precise measure of the nearest millimeter. This was more difficult for them. Some could not do it. A tolerance of one millimeter should be allowed for sixth and seventh grade pupils.

After the relation of the centimeter to the millimeter and the reverse had been established an exercise in changing centimeters to millimeters and the reverse was given. This offered no difficulty.

The next exercise was the important one in this lesson. The pupils were asked to find 11 millimeters on their rulers. Then they were asked to read and write the 11 millimeters as centimeters. This they did as 1.1 centimeters. Then, *and here is the crucial step*, they were led to see that the right hand 1 in 11 mm. is one tenth of the left 1 because in reading the identical 1.1 centimeters the right 1 is worth one tenth of the left 1 by the very act of reading it as *1.1 centimeters* and reinforced by the further act of seeing the 1 millimeter as .1 centimeter right on the ruler before them. Practice with other numbers on the ruler, care being taken that the two figures in the numbers were the same—as 55 mm. =5.5 cm. and 9.9 cm. =99 mm.—established pretty well the place value concept as far as numbers on the ruler went.

In the next exercise numbers in United States money were used such as 11 cents and 44 cents. Pupils were asked to go to the board and show or prove that the left figure in the above is worth ten times the value of the right figure and that the right figure is worth one tenth of the value of the left figure. Then three-place numbers were taken up such as $3.33 and pupils were asked to prove that the first figure is worth ten times as much as the second and one hundred times as much as the third and the reverse steps. Then other numbers were used such as 555 children in the same way and then abstract numbers were introduced consisting of three and four figures such as 4,444 and volunteers showed and proved the relations of every 4 to every other 4 both to the right and left. Not all but about half of the class were able to do this.

As a final stage in the lesson, numbers of different figures such as 42 and 24 were introduced. By a series of questions and illustrations it was shown that since the 4 by itself is worth two times the 2 by itself, the 4, now, since it stands in a place which has ten times the value of the place in which the 2 stands, has really twenty times the value of the 2 in the number 42. Similarly it was shown that the 2 in 24 has only five times the value of the 4. A final series of questions on what is the relative value of the different figures in a number such as 2,048 brought out further meanings. Some of these meanings were grasped by only a few in the class depending upon their maturity.

The lesson was concluded by having the members of the class write in their notebooks what they had discovered, namely, *the figures in any number of our number system have two values: one value is that which the figure has when standing by itself,*

the other, called its place value, comes from the place it occupies in the number and this value is always ten times the place value of the figure next to the right.

Two class periods of 45 minutes each were used in teaching this lesson.

To ascertain how well the pupils had learned the concept of place value, within a week after the teaching, a 45-minute period was used in giving the class a 20-item test. There were 120 pupils in the four classes of 6B, 6A, 7B and 7A from Chicago schools. Their average C.A. was 12 years, 1 month, with range of 3.4 years; the average M.A. was 12 years, 11 months, and range of 5.8 years; the average I.Q. was 106.8 with range of 58 (76-134).

In the results of the test the median score was 70%, the first quartile was 60% and the third quartile, $82\frac{1}{2}$%.

The first four questions asked to measure the length and width of a card given to each member of the class, first to the nearest centimeter and then to the nearest millimeter. About 63% of the class had correct answers to these four. The next eight items asked for changing centimeters to millimeters and the reverse both on and off the ruler. To these eight, 94% gave correct answers. Questions 14, 15, 16 and 18 really asked for an understanding of the meaning of place value of any figure in different numbers of two, three and four similar figures including abstract numbers. The questions asked the pupil to show or prove that any figure was worth ten times the value of figure next to right or one tenth of value of figure next to the left. A little less than one half (44%) of the pupils showed ability to do this.

So far the test had dealt with numbers of like figures only. Question No. 19 read: "In the number 63, how many times the value of the 3 is the value of the 6?" This, as is seen, is a more difficult question. 60 out of the 120 answered correctly to this question. Question 20 read, "In the number 105, what part of the value of the 1 is the value of the 5?" An answer to this question would confirm the understanding of place value. The answer, 1/20, was given by 37 out of the 120 or 31%.

The writer was interested in finding what the relation was between M.A.'s and I.Q.'s on the one hand and the results from the test. The median M.A.'s and I.Q.'s were found. The relations are shown below.

	Lower half	*Upper half*
Median raw score	12	$16\frac{1}{2}$
Median M.A.	12 yrs. 1 mo.	12 yrs. 11 mo.
Median I.Q.	103	110

	First Quarter	*Second Quarter*	*Third Quarter*	*Fourth Quarter*
Median score	$10\frac{1}{2}$	$13\frac{1}{2}$	15	18
Median M.A.	12—0	12—6	12—8	13—2
Median I.Q.	101	104	106	115

There is no doubt that if more time had been spent on teaching this lesson better results would have been obtained. This will offer an invitation for others to do so.

It does point to the fact, however, that the meaning of place value in our arithmetic is more difficult to understand than has been realized perhaps, but that a child of mental age of 12 years and 6 months is sufficiently mature to grasp an understanding of place value and that further efforts should be made in devising lessons and tests for a better understanding of place value by pupils.

The Performance of Academic and Vocational High School Pupils on the Cooperative Mathematics Test

JOSEPH JUSTMAN *and* GEORGE FORLANO
Bureau of Educational Research
Board of Education of the City of New York

IT IS commonplace in educational thinking to accept, without reflection, the observation that academic high school students enrolled in courses in mathematics will show better achievement than pupils completing the same basic courses in a vocational high school. Such comparisons, of course, do not take into consideration the basic inequality in intellectual status which characterize the two groups of students. Any approach to a consideration of the relative achievement of academic and vocational school pupils should, if one is to arrive at valid conclusions concerning the effectiveness of student mastery of skills and knowledges in mathematics, be based upon a comparison of pupils of equivalent intellectual ability.

As one aspect of a comprehensive survey of the effectiveness of instruction in vocational education,[1] conducted under the auspices of the New York State Department of Education, the Cooperative Mathematics Test for Grades 7, 8, and 9, Form Q, was administered to 190 matched pairs of eighth term pupils enrolled in several academic and vocational high schools. The academic and vocational high school students were equated on the following bases: MA, CA, IQ, sex and school grade.

The Cooperative Mathematics Test, in addition to a total score, also provides for the determination of subscores in each of the following areas: Skills, Facts, Terms, and Concepts, Applications, and Appreciations. It was felt that the Cooperative Mathematics Test might serve to measure those aspects of performance in mathematics ordinarily subsumed under the heading of "general education."

Table I presents the mean raw scores of the 190 matched pairs of pupils enrolled in the participating academic and vocational high schools on the total test and on each subtest. In addition, the table summarizes the results of testing the observed mean differences between the two groups for statistical significance.

Inspection of Table I reveals that the group of pupils drawn from the academic schools shows slightly superior performance on each of the subtests as well as on the total test than does the matched group of pupils drawn from the vocational schools. An interesting phenomenon appears, however, when the differences between groups are tested for significance.

Such analysis indicates that, while the total score of the group of academic school pupils is significantly higher than that of the matched group of vocational school pupils, this superiority may be attributed, in large measure, to the results of Part IV of the Cooperative Mathematics Test. This subsection of the total test measures, for the most part, pupil appreciation of the mathematical process involved in the computations that he performs and the problems that he solves. It is not surprising to find, in view of the greater stress in the academic schools upon what might be considered more

[1] J. Wayne Wrightstone, Robert Egbert, George Forlano and Joseph Justman. *Measuring the Effectiveness of Instruction in Vocational Education*, Board of Education of the City of New York, Bureau of Education Research, October 1950, 149 pp. (Mimeographed)

TABLE I

Mean Scores and Significance of Mean Differences of Cooperative Mathematics Test Scores of Participating Vocational and Academic School Students

Group	No.	Mean	Standard Deviation	Mean Diff.	P†
Part I Skills					
Academic	190	26.21	8.55	1.37	$.10>P>.05$
Vocational	190	24.84	7.65		
Part II Facts, Terms and Concepts					
Academic	190	17.97	5.70	1.00	$.10>P>.05$
Vocational	190	16.97	5.65		
Part III Applications					
Academic	190	16.55	6.05	.44	$.50>P>.40$
Vocational	190	16.11	6.40		
Part IV Appreciations					
Academic	190	13.84	4.60	1.26	$P<.01$**
Vocational	190	12.58	4.90		
Total Test					
Academic	190	74.07	21.80	4.73	$.05>P>.02$*
Vocational	190	69.34	21.60		

* Significant.

** Highly Significant.

† The P value represents a statistical evaluation of the probability that an obtained mean difference is a reliable one and cannot be attributed to chance factors. For example, if $P<.01$, the probability that such a difference would occur by chance is less than 1 out of 100, and therefore, the mean difference is considered reliable. Ordinarily a P value of .05 is looked upon as the lowest value delimiting a reliable difference.

theoretical formulations, that such varying curricular emphases are reflected in test results. In spite of the slightly higher scores earned by academic pupils, none of the other subsections of the total test reveal any significant differences. Particularly worthy of note is the fact that on that section of the total test dealing with application of mathematical skills and knowledges, an extremely small and insignificant difference between the two groups was obtained.

The indications are, then, that academic and vocational high school students who are equivalent in intellectual ability will show only minor differences in achievement as measured by the Cooperative Mathematics Test. However, the academic pupil appears to function at a higher level in those aspects of the subject centering about appreciation of mathematical processes. Greater emphasis on the appreciation phase of mathematics in the vocational high school curriculum would probably serve to reduce the differences revealed by the present study.

The Institute for the Unity of Science is offering a prize of $500 for the best essay on the theme "Mathematical Logic as a Tool of Analysis: Its Uses and Achievements in the Sciences and Philosophy." Two additional prizes of $200 each will be given for the next best two essays. This is an international contest and is open to everyone. Essays must not exceed 25,000 words. They may be written in English, French, or German, and must be submitted before January 1, 1953. The winners of the previous essay contest sponsored by the Institute were Dr. George Kline of Orangeburg, N. Y., and Mr. Eugene E. Luschei of Lincoln, Neb. Each of them received a second prize of $150. No first prize was awarded. Further information on the contest can be obtained from the Institute for the Unity of Science, American Academy of Arts and Sciences, 28 Newbury Street, Boston 16, Massachusetts.

The 1952 annual meeting of the **Duodecimal Society of America** was held in New York on January 24th. Dr. C. T. Dieffenbach, visiting lecturer at the Paterson (N.J.) State Teachers College, described his uses of the duodecimal system—counting by dozens—as a device in teaching teachers. Technical papers on primitive right triangles were presented by Robert S. Beard, Colonel of Army Engineers (retired), and Ralph H. Beard, secretary of the Duodecimal Society and a service engineer of the New York Telephone Company. Kingsland Camp, of the Equitable Life Assurance Society, spoke on duodecimal relationships of time, the stars, and the calendar.

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

49. Pythagorean Numbers

The following letters are reproduced for the testimony they bear to (a) the "intellectual" and "avocational" interests which a surprising number of non-professional mathematicians can and do have in our subject, and (b) the excellent independent thinking which such persons (including some of our students) can do with some stimulus.

DETROIT, MICHIGAN
October 20, 1951

DEAR DR. JONES:

I have been interested in your "Mathematical Miscellanea" in THE MATHEMATICS TEACHER and note especially two articles concerning the Pythagorean theorem.[1] A friend of mine (not a teacher) got so interested in the formula $a^2+b^2=c^2$ that he evolved formulas for the relationship of a, b, c. To wit:

When a is odd,

$$b=\frac{a^2-1}{2} \tag{1}$$

$$c=\frac{a^2+1}{2}$$

When a is even,

$$b=\left(\frac{a}{2}\right)^2-1$$

$$c=\left(\frac{a}{2}\right)^2+1. \tag{2}$$

These values will check. When a is odd:

$$c+b=a^2$$
$$c-b=1$$
$$c^2-b^2=a^2$$

When a is even:

$$c+b=\frac{a^2}{2}$$
$$c-b=2$$
$$c^2-b^2=a^2.$$

[1] *See* THE MATHEMATICS TEACHER, XLIII (April 1950), 162–63; (Nov. 1950) 352.

The first set (a odd) goes back to the time of Pythagoras; the second set (a even) I'm not so sure about, but I think to Proclus. The validity of the formulas can be demonstrated by innumerable examples:

a	b	c
1	0	1
2	0	2
3	4	5
5	12	13
7	24	25
.	.	.
.	.	.
.	.	.
20	99	101
.	.	.
.	.	.
.	.	.
33	544	545

(and multiples of these numbers).

However, some values of a admit of other values for b and c than those expressed by the above formulas. For example:

a	b	c
20	21	29
30	56	65

This disturbed my friend until he came up with the following offering:

If $a=s\cdot r$, where $s<r$,

Then, when a is odd

$$\left(\frac{a}{s}\right)^2=c+b, \qquad \left(\frac{a}{r}\right)^2=c-b, \tag{3}$$

when a is even,

$$2\left(\frac{a}{s}\right)^2=c+b, \qquad \frac{\left(\frac{a}{r}\right)^2}{2}=c-b\cdot \tag{4}$$

This second pair of formulas will check also. Check:

$$c+b=\left(\frac{a}{s}\right)^2$$

$$c-b=\left(\frac{a}{r}\right)^2$$

$$c^2-b^2=\frac{a^4}{s^2r^2}$$

or $c^2-b^2=a^2$ and

$$c+b=2\left(\frac{a}{s}\right)^2$$

$$c-b=\frac{\left(\frac{a}{r}\right)^2}{2}$$

$$c^2-b^2=\frac{2\,\frac{a^4}{s^2r^2}}{2}=a^2$$

or $c^2-b^2=a^2$.

Using each set of formulas on any given value of a seems to exhaust all possibilities for the relation $a^2+b^2=c^2$. Can you verify or nullify this?

Very truly yours,
CLARA H. MUELLER
Cass Technical High School

University of Michigan
ANN ARBOR, MICHIGAN

DEAR MISS MUELLER:

Thank you very much for the note from your friend. It is nice to know that people do have fun with mathematics. There is only a little to be added to the excellent analysis which you two have given.

Historically, as you stated, formulas (1) are attributed to Pythagoras. They hold only for a odd. Proclus attributed a second set of formulas to Pythagoras;[2] namely,

(5) $x=2n+1,\ y=2n^2+2n,\ z=2n^2+2n+1,$

which, you will note, are not really different from what your friend states in (1) for they require x to be odd, and b and c to differ by 1, and, in fact, reduce to your friend's formulas by substituting $n=x-1/2$ in y and z.

Plato has been given credit for the formulas $x=2m,\ y=m^2-1,\ z=m^2+1$.[3] In this case x is even, y and z differ by 2, and your friend's formulas (2) may be derived by substituting $m=a/2$.

Euclid is credited with being the first to give general formulas which will produce all sets of Pythagorean integers. Of course, none of the early Greeks wrote formulas as we have here, but rather used an involved geometric terminology. Simplified and put into modern notation the general formulas are:

(6) $x=p^2-q^2,\quad y=2pq,\quad z=p^2+q^2,$

where p and q are relatively prime (otherwise, x, y, z would still be a Pythagorean set, but not a *primitive* one). Note that if p and q are relatively prime x and z are both odd and y is even.

All of the formulas cited so far are special cases of the general ones; for example, we get (5) if $p=n+1,\ q=n$. However, your friend's equations (3) and (4) are not merely special cases but are equivalent to (6) if taken together and if s and r, the factors of a are regarded as parameters, rather than a. For, if a is odd and composite, one may always let $a=p^2-q^2=(p-q)(p+q)$. If you now let $s=(p-q)$, $r=(p+q)$ it's easy to show that your friend's equations lead to $c=p^2+q^2$ and $b=2pq$, or to (6).

Similarly, if a is even, let $a=s\cdot r$ where $s=2p$ and $r=q$. It then follows that formulas (4) are equivalent to (6).

This implies the answer to your final question; namely, it *is* true that formulas (3) and (4), with (1) and (2) regarded merely as special cases, actually do exhaust all the possibilities for $a^2+b^2=c^2$.

I have made this final conclusion hinge upon (6) because there are several interesting proofs of it. Oystein Ore gives one which has elements in common with your approach;[4] Felix Klein gave one using a point lattice and analytic geometry procedures.[5]

I hope all this has been fun for you and your friend as it has for me. Send me more of your good mathematical gleanings.

Sincerely yours,
PHILLIP S. JONES

50. An Unanswered Letter

The recent publication of two letters and their answers in this Department (*Miscellanea 46 and 49*) has led to the suggestion that our readers might enjoy sharing in and even improving upon the answers if the letters alone were published. Let's try it! Here is a letter. What would your answer be? Have you a letter for which you would like us to solicit answers?

1752 Long Valley Road
GLENVIEW, ILLINOIS
November 30, 1951

DEAR SIR,

I am 13 years old and a freshman at Niles Township High School. Recently I was talking with my algebra teacher, Miss de Booy, about numbers. When I told her I was at a dead end concerning information, she suggested that I write to you.

My main interest, is to find the highest actual number known to man. Already I have progressed farther than many sources of information but I have done so only by repetition. After writing the Library of Congress and being advised to go to the Chicago Public Library, and

[2] Oystein Ore, *Number Theory and Its History.* (New York: McGraw-Hill Book Co., 1948) p. 166.

[3] T. L. Heath, *A Manual of Greek Mathematics.* (Oxford: Clarendon Press, 1931) pp. 47, 48.

[4] Ore. *op. cit.*, p. 167.

[5] Felix Klein, *Elementary Mathematics from an Advanced Standpoint, Arithmetic-Algebra-Analysis.* (New York: Dover Publications, 1945) pp. 44–46.

there finding nothing I have lost faith in that source.

Could you possibly give me information about means of accomplishing my goal? Thank you.

Yours very sincerely,
TOM HAWK

51. *Dictionary Delving*

I recently received a query as to where one could find an easily available and authoritative single source for the pronounciation of mathematical terms and especially of the names of mathematicians of historical interest. I could give no answer, especially for the latter part of the question. Can you?

This suggested that perhaps this Department could serve as an exchange of questions and answers of this type. What do you think?

Of course, everyone has access to dictionaries and encyclopedias. The function of such notes might then be: (1) to note meanings and changes of meanings which either are not recorded by nontechnical references or which students, unaware of their existence, might not ever even try search out, (2) to record pronounciations of names or terms which are not easily found or whose pronounciations have changed, (3) to point out etymological or historical backgrounds for words which add interest and often insight and meaning to the teaching of related mathematical topics.

As illustrations I comment here on four words which are in everybody's dictionary, but which typify the problems and interest which a dictionary may provide for students of mathematics. Do you like this idea? If so send us your word questions, problems, pet peeves, sources of information.

PY-THAG′-O-RE′ AN: This word which today is properly pronounced with a short *i* sound in the first syllable and the primary accent on the next to the last syllable[6] was first spelled and pronounced *Py-tha-go′-ri-an* according to the *New English Dictionary*. The spelling changed c. 1600 but the old pronounciation was still used by Cowley and Dryden. Non-mathematical uses of the word include *p. bean*, *p. comma*, *p. letter*, *p. lyre*, *p. scale*, *p. semitone*, *p. third*, *p. system* (of astronomy).[7]

Mathematical uses include *p. identities*, *p. numbers*, *p. relation between direction cosines*, *p. theorem*.[8]

AB′-A-CUS (abaci)[9]—or isn't it often mispronounced in your part of the country?

CON′-GRU-ENT. I have a hard time resisting an impulse to accent the *gru*.

James and James' definition, "Figures which can be superposed (placed one upon the other) so that they coincide, i.e. they are equal in all their properties,"[10] involves all the difficulties of superposition which were pointed out in *Miscellanea 46*. Mathematically, *congruence* in many geometric systems, e.g. Hilbert's[11] is an undefined term or relation which gains its meaning implicitly from the assumptions made about it in a set of axioms. Dictionaries, however, don't recognize the existence of undefined terms, apparently. Should they? How?

Another dictionary definition suggests the difficulty which I once saw almost disrupt a freshman class. Are two triangles congruent if their equal angles and sides

[6] Glenn James and Robert C. James, *Mathematics Dictionary*. 2nd ed. (New York: D. Van Nostrand Company Inc., 1949). This is the only mathematics dictionary in English in print known to the writer.

[7] *A New English Dictionary on Historical Principles*. (Oxford University Press, 1909), vol. VII. This is a fascinating source. It contains dated and documented quotations showing the first uses of a word in each of its different meanings. A much condensed two volume edition has been published more recently.

[8] James and James., *op. cit.*

[9] *Ibid.*

[10] *Ibid.*

[11] David Hilbert, *The Foundations of Geometry*. (Chicago: Open Court Publishing Co., 1902). For a further technical discussion of congruence and rigorous approaches to it see Federico Enriques, "Principles de la Géométrie," in *Encyclopédie des Sciences Mathématiques Pures et Appliquées*. Tome III, Vol. i, Fascicule 1, p. 27 or the article by A. Guarducci "Della Congruenza e del Movimento" in Enriques' *Questioni Riguardanti Le Mathematiche Elementari*. (Bologna, 1924), pp. 109–142.

are differently ordered? What about polyhedral angles and tetrahedra? *Webster's New International Dictionary* (2nd ed., G. and C. Merriam Co., 1934) page 563 says under *congruent.* "Superposable so as to be coincident throughout. Two plane figures are *directly congruent* when they can be brought into coincidence without the removal of either from their common plane, and two space figures when they may be brought into coincidence with each other by a rigid motion. Two plane figures are *inversely congruent* when they can be brought into coincidence only by the removal of one of the figures from the common plane, and two space figures when one of them is directly congruent with the reflection of the other in a plane."

Do we teach this? Should we?

Other mathematical uses of the word congruent are for: (1) numbers leaving the same remainder on division by the same integral modulus, (2) a linear fractional substitution (transformation) in the theory of discontinuous groups, (3) certain pairs of matrices.

FRACTION is derived from the Latin *frangere, fractus,* to break. Fractions have in fact also been called broken numbers and in German *gebrochene zahlen.* The facts that the Arabic word for fraction, *al-kasr,* is derived from the stem of the verb meaning *to break,* and that we have an English word *fracture* provide historical background and current connections that add interest and meaning to teaching.[12]

52. The Commutative and Associative Laws of Addition[13]

We have all taught the commutative and associative laws of addition and felt, perhaps, that the fact that they are assumptions was lost upon the children. It seems so obvious that $2+3+4$ equals $3+4+2$ or that $(2+3)+4$ equals $2+(3+4)$ that the young student does not see what all the fuss is about. One can point out that the law of order does not hold in subtraction, but this is all rather stuffy. The magnitude of the assumptions, their generality, and their role in the realm of numbers can be brought home by concocting instances in life where the laws lead to ridiculous situations. Good Professor Dresden used to point out that one could put on his hat first and then his coat or the coat first and then the hat and it made little difference, although I am told by some women friends that the hat certainly should come first. But there is no dispute over whether stockings should precede shoes. Boys and girls get the trick quickly and enjoy imagining such examples as these:

1. Add sulphuric acid to water;—harmless.
   Add water to sulphuric acid;—tremendous heat, possible explosion.
2. Cooking is full of examples. Do not put baking powder into milk; mix it with the last cup of flour. When making lemon pudding, put the lemon juice into the egg yolks and then add milk. If milk is put into lemon juice, it curdles.
3. If you have tickets for the football game beginning at 2 P.M. and the opera at 8:30, it makes a big difference whether you go to the opera first or second. If you try going to the stadium second, you may have the choicest seats on the fifty yard line all to yourself in the darkness.

(Continued on page 275)

[12] L. C. Karpinski, *The History of Arithmetic* (Chicago: Rand McNally & Co., 1925), p. 126. D. E. Smith and Jekuthiel Ginsburg, *Numbers and Numerals,* (New York: Bureau of Publications, Teachers College, Columbia University, 1937), p. 49. Both of these works have chapters devoted to the stories of some arithmetic words. The latter booklet is now available from the Washington, D. C. headquarters of the National Council of Teachers of Mathematics.

[13] *Department Editor's Note:* Further interesting and more technical discussions of these topics are to be found in Mannis Charosh "Unifying Elementary Mathematics by Means of Fundamental Concepts," *National Mathematics Magazine,* XIX (1944), p. 78 ff. and E. B. Mode "The Commutative Law," THE MATHEMATICS TEACHER, XXXVIII (1945), p. 108 ff.

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RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
School of Education, New York University, New York 3, N. Y.

THE PROBLEM OF LOCATING RESEARCH STUDIES IN MATHEMATICS EDUCATION

Despite the fact that there are several general publications[1] designed to help students find educational information and data, communications reaching this department seem to indicate a need for dealing with the specific problem of locating, with a minimum of time and energy, studies in mathematics education. What follows is merely a distillation of one individual's experience; other suggestions from readers will not only be welcomed but will be reported in future issues.

The problem is becoming more complex because investigators are becoming more sensitive to the fact that learning and curriculum making in mathematics are related to many other fields. The psychology of learning is a factor, for example, in the emphasis on meanings, teaching for transfer in the case of geometry and in the development of concepts and generalizations. Studies on the uses of visual aids in social studies and science education have some implications for the teaching of mathematics. Philosophical studies and analyses have something to contribute to revealing the values accepted when functional mathematics and usage studies dominate the construction of mathematical curricula. Recent research studies in statistics must be known if conclusions drawn from comparisons of methods of teaching mathematics, for instance, are to weight properly the many variables contributing to the results obtained.

For over-all summaries of research the articles dealing with arithmetic and secondary school mathematics in the 1941 and 1950 *Encyclopaedia of Educational Research*, published by the American Educational Research Association with W. S. Monroe as editor, are extremely helpful. The bibliographies appended to each article provide more specific aid. This same Association, with headquarters at 1201 Sixteenth St., N. W. Washington 6, D.C., also sponsors the periodical *The Review of Educational Research*. Some issues of this publication dealing with elementary and secondary school mathematics are as follows: Vol. I, Dec. 1931, pp. 261–275 and 367–370; vol. II, Feb. 1932, pp. 7–20; vol. IV, April 1934, pp. 140–143, 168–171 and Dec., pp. 479–488; vol. V, Feb. 1935, pp. 14–30; vol. VII, April 1937, pp. 160–162 and Dec., pp. 453–463; vol. VIII, Feb. 1938, pp. 51–57; vol. XII, Oct. 1942, pp. 356–404, 405–411 and 425–434; vol. XV, Oct. 1945, pp. 276–288, 298–300 and 310–320; vol. XVIII, Oct. 1948, pp. 337–349 and 350–363; and vol. XXI, Oct. 1951. The current plan is to report every three years in October research completed in the fields of mathematics and science, and described in periodicals and pamphlets especially. Most doctoral dissertations and even some masters' theses are reviewed. Among others, single copies of the October 1951 and October 1948 issues are still available at this writing for $1.50 and $1.00 respectively. During 1939 this same organization published "The Implications of Research for the Classroom Teacher." In chapter 10 of this booklet G. T. Buswell summarized important research in arithmetic and H. E. Benz, in secondary school mathematics.

The National Council of Teachers of

[1] For example, Alexander Carter. *How to Locate Educational Information and Data.* (New York: Teachers College, Columbia University, 1941.)

Mathematics through its yearbooks and its journal, THE MATHEMATICS TEACHER, has, quite naturally, done outstanding work in reporting research in the curriculum and teaching of mathematics. The index for the year is usually found in the December issue of the journal. Some of the yearbooks which seem especially important to research are the First, Second and Fourth for their historical and comparative surveys of methods, curriculum and teacher training, along with the Fourteenth. The Ninth, by H. R. Hamley, is very significant to those interested in the development of functional thinking in mathematics. The Thirteenth, by H. P. Fawcett, is the famous *Nature of Proof*. In the Eighth, the Teaching of Mathematics in the Secondary School, H. E. Benz summarized on pages 14–54 the research done from about 1915 through 1931. He followed this with later reviews in THE MATHEMATICS TEACHER for October 1933 (vol. 26, pp. 372–381) and November 1934 (vol. 27, pp. 344–348). The Tenth, *The Teaching of Arithmetic*, and the Sixteenth, *Arithmetic in General Education*, contain research reports, reveal the development of the current "meaning" emphasis in the teaching of arithmetic. The Sixteenth has chapters on evaluation and psychology of learning arithmetic interlaced with research studies and a final section listing 100 important researches in arithmetic curriculum and methods. Although many of the yearbooks are now out of print, they can be found in the libraries of most educational institutions. THE MATHEMATICS TEACHER occasionally prints abstracts of doctoral dissertations and in rarer instances has reproduced entire theses in serial issues.

The National Society for the Study of Education has published, among others, yearbooks entitled *Research in Arithmetic* (1930), *Scientific Method in Education* (1938), *The Psychology of Learning* (1942), *Learning and Instruction* (1950) and *The Teaching of Arithmetic* (1951). These scholarly efforts involving the reporting and interpreting of research cannot be safely ignored by the careful investigator in mathematics education.

Books on the teaching of mathematics which have unusually strong and up-to-date bibliographies are *The Teaching of Mathematics* by C. H. Butler and F. L. Wren (McGraw-Hill, 1951) and *Teaching the New Arithmetic* by G. M. Wilson (McGraw-Hill, 1951).

For locating periodical references in general *The Education Index* is superior in its field. *The Readers Guide to Periodical Literature* and *The International Index* are important supplements. Mention at this point must be made of *A Bibliography of Mathematical Education* by William L. Schaaf (Stevinus Press, Forest Hills, N. Y., 1941). This contains a list of approximately 4,000 periodical references for the period 1920 to 1941. By means of it pertinent references can be located much more rapidly than with the more general indices. The National Council, in this writer's opinion, ought to induce someone like Dr. Schaaf to produce a similar bibliography for the ten year period since 1941.

In addition to THE MATHEMATICS TEACHER some of the other periodicals which more or less frequently report research relating to mathematics education are *School Science and Mathematics*, *Journal of Educational Research*, *Journal of Experimental Education*, *Education Abstracts*, *Education Digest*, *High Points*, *California Journal of Secondary Education*, *Elementary School Journal*, *Educational Research Bulletin*, *Journal of Educational Psychology*, *Peabody Journal of Education*, *Psychological Abstracts and Psychological Bulletin*. *The School Review* gives brief annotations on studies in various fields. *The Elementary School Journal* does a similar task in the field of arithmetic at least once a year.

Through interlibrary loans it is possible to procure copies of doctoral dissertations. Each year *Doctoral Dissertations Accepted by American Universities* (H. W. Wilson Co., New York) is published under the

editorship of Arnold H. Trotier. The *Phi Delta Kappan* lists annually in its February issue "Doctors' Dissertations Under Way in Education." This same journal publishes each September a "Research Methods Bibliography" by Carter V. Good. Several universities publish abstracts of doctoral dissertations. Until 1941, the United States Office of Education published a bulletin annually (usually number five) entitled "Bibliography of Research Studies in Education" which gave brief annotations of masters' theses and doctoral dissertations.

Teachers College, Columbia University in its series *Contributions to Education* published until recently nearly all of its Doctor of Philosophy dissertations. As a result, in mathematics education these studies are, no doubt, the most available and best known of any in the United States.

The writer would be remiss if he did not pay tribute to the University of Chicago and the University of Illinois for their work in summarizing research in the field of arithmetic in earlier years. G. T. Buswell and C. H. Judd prepared the *Summary of Educational Investigations Relating to Arithmetic* (Supplementary Educational Monographs, No. 27, University of Chicago Press, 1925) followed by two later supplements. In the same field W. S. Monroe and M. D. Engelhart contributed *A Critical Summary of Research Relating to the Teaching of Arithmetic* (University of Illinois, Bulletin, No. 5, Urbana, Ill., 1931) and the former with others also wrote *Ten Years of Educational Research, 1918–1927* (Bureau of Educational Research, Bulletin, No. 42, University of Illinois, Urbana, Ill., 1928).

Finally, for those whose research demands the use of formal tests and other means of evaluation the *Mental Measurement Yearbooks* (1938, 1940, 1950) by Oscar K. Buros of Rutgers University provide critical evaluation of nearly all instruments of appraisal in the fields of education and psychology.

Mathematical Miscellanea

(*Continued from page 272*)

4. One may take a bath and then put on his clothes. It does not work so well the other way around!

The associative law has equal possibilities for dubious illustration.

1. A house is built with *foundation* $+$(*nails* $+$ *wood*)
   It will not do to take (*foundation* $+$ *nails*) $+$ *wood*.
2. (*Hot dogs* $+$ *mustard*) $+$ *ice cream* is good fare but do not try *Hot dogs* $+$(*mustard* $+$ *ice cream*).
3. *John* $+$ (*Susie* $+$ *Tom*) go to the dance or (*John* $+$ *Susie*) $+$ *Tom* go. They all get there, but everyone knows it makes a big difference who takes Susie and who goes stag.

These are enough to start one on his own way to finding humor in the commutative and associative laws. That he laughs at the antics of these laws does not lessen his comprehension of them, and the fun may help fasten these principles in mind. As the student recognizes the many instances where they are utterly inappropriate, the fact that they are true for numbers becomes the more amazing. He begins to realize the significance of that word *assumption;* that there really is a lot to swallow without proof. And out of this contemplation grows a new respect for the nature of the number system, and a challenging and questioning frame of mind that leads one to the very character of mathematics.

(Miss) A. N. Tucker
J. Sterling Morton High School
Cicero, Illinois

APPLICATIONS

Edited by SHELDON S. MYERS
Department of Education, Ohio State University, Columbus, Ohio

SEVERAL MONTHS ago John Schacht of Bexley, Ohio, who has been pioneering with flexible geometry models for well over a decade, contacted me and inquired whether I knew of an application of Ceva's Theorem. This theorem was published by Giovanni Ceva at Milan, Italy, in 1678 and may be briefly stated as follows: If three concurrent lines are drawn from the vertices of a triangle to the opposite sides dividing the sides into pairs of segments, the product of three non-adjacent segments is equal to the product of the other three segments.

The first idea that came to me was the method of representing three-component phase diagrams in physical chemistry. Upon investigation, this example turned out to be an illustration of another theorem of geometry and not Ceva's Theorem. Mr. Schacht and I independently brought Clarence Heinke of Capital University, Columbus, Ohio, into our quest. It occurred to Mr. Heinke that Ceva's Theorem was an example of joint variation and could be used to geometrize any problem involving joint variation. Mr. Heinke's idea resulted from contemplating the following problem: If 7 cats can kill 7 rats in 7 minutes, how many cats would be needed to kill 100 rats in 50 minutes? The solution of this problem can be symbolized as follows:

Let r = number of rats, c = number of cats, t = number of minutes.

Assuming, $r \sim c$ and $r \sim t$.

When a variable is proportional to two variables jointly, it is proportional to their product:

$$r \sim ct$$

or

$$\frac{r_1}{r_2} = \frac{c_1 t_1}{c_2 t_2}$$

whence, $r_1 c_2 t_2 = r_2 c_1 t_1$.

The problem can now be solved either by algebraic substitution or by Ceva's Theorem. In the latter method, r_1, c_2, t_2, are made to represent one set of three non-adjacent segments (see statement of theorem above), while r_2, c_1, and t_1 represent the other set.

P.G. 6 Gr. 10–14 *Use of Ceva's Theorem with Two Practical Problems Involving Joint Variation*

There are many possibilities for problems involving joint variation. One of these suggested by Mr. Heinke, the general gas law, is quite appropriate for high school. Here is a typical problem involving this law:

A gas under 1 atmosphere pressure at 0° Centigrade has its temperature raised to 273.1° Centigrade and its pressure raised to 6 atmospheres. If its initial volume was 20 liters, what will be its final volume?

The general gas law is usually stated in high school texts as follows:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}.$$

For Ceva's Theorem this can be rewritten as follows:

$$P_1 V_1 T_2 = P_2 V_2 T_1$$

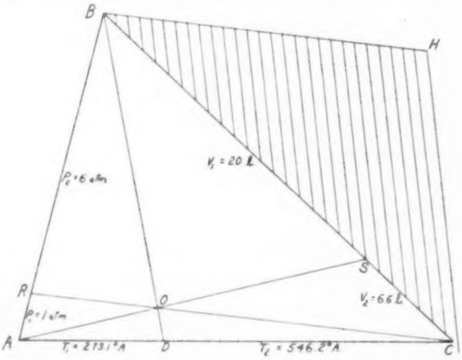
where T is in Absolute degrees.

(The Absolute degrees can be found simply by adding 273.1° to the Centigrade degrees.)

The data of the problem can be organ-

ized as follows: $P_1 = 1$ atm.; $P_2 = 6$ atm.; $T_2 = 546.2°$ A; $T_1 = 273.1°$ A; $V_2 = ?$; $V_1 = 20$ liters.

The solution by Ceva's Theorem is shown in the drawing that follows:



Ceva's Theorem and the General Gas Law

A most fascinating exercise in functional visualization can be performed mentally with the above figure. For instance, if the line BD were a median, the physical change taking place in the gas would be an *isothermal* one, meaning constant temperature throughout the change. If AS were a median, the change would be called an *isometric* one, meaning constant volume. If RC were a median, the change would be called an *isobaric* one.

Assuming BD to be a median, the relationship exhibited between the pressures and volumes in the above diagram is none other than the well-known "Boyle's Law." Visualize BD fixed as a median. Now imagine that the final pressure is less. This would have the effect of moving R toward B as well as moving the point of concurrency O toward B. If O moved toward B, then S would move toward B, thus increasing the final volume. Consequently when the final pressure is less, the final volume is greater, which is what is meant by "varies inversely."

When RC is assumed to be a median, the relationship exhibited between volume and temperature is none other than the "Law of Gay-Lussac and Charles." Visualize RC a median. Now imagine the final temperature to be increased. This would have the effect of moving D toward A as well as moving the point of concurrency toward R. If O moved toward R, then S would move toward B, thus increasing the final volume. Since the final volume increases as the final temperature increases, this is what is meant by "varies directly."

Note that the units of pressure and temperature are selected arbitrarily on the sides AB and AC at the beginning. However the position of S is not known until AS is drawn through the point of concurrency O. Consequently the length of segment BS representing 20 liters is not known until AS is drawn. Thereupon the unit of length on BC representing one liter can be found, after BS is known, by dividing BS into 20 equal segments. This explains the use of the auxiliary construction line BH. The length of BC in the beginning can be any size, while the lengths of AB and AC are determined by the assumed lengths of units of pressure and temperature respectively.

Later, after seeing Mr. Heinke's examples, Mr. Schacht suggested the following:

If 4 men can build 2 prefabricated houses in 6 hours, how many houses could 12 men build in 4 hours?

Let m = number of men; p = number of prefabs; h = number of hours. Assuming,

$$p \sim m \quad \text{and} \quad p \sim h$$

$$p \sim mh$$

$$\frac{p_1}{p_2} = \frac{m_1 h_1}{m_2 h_2}$$

$$p_1 m_2 h_2 = p_2 m_1 h_1.$$

Another problem could be:

If a 24 horsepower engine uses 10 gallons of fuel in 8 hours, how many gallons would a 50 horsepower engine use in 5 hours?

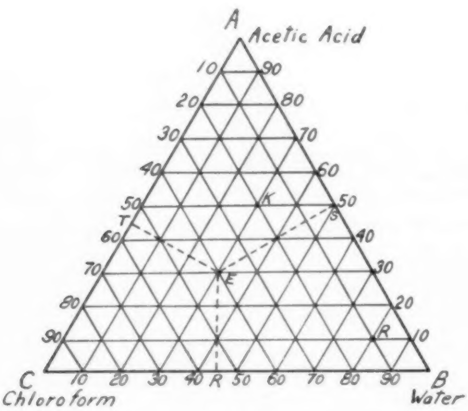
Here again it is necessary to assume that fuel consumption is directly proportional to horsepower and time.

P.G. 7 Gr. 10–14. *Use of a Geometry Theorem in Three-Component Phase Diagrams.*

It may be a matter of interest to present the application which I found while on

the "Ceva quest."

In physical chemistry and physical metallurgy it is convenient to represent mixtures of three chemical substances, known as components, by means of a diagram in the shape of an equilateral triangle. Each vertex of the triangle stands for one of the three pure components and represents 100% of the component. Any point within the triangle represents the percentages of the three components in a given mixture. Each percentage is measured by the perpendicular from the point to a side. Thus, in the figure below, for point E, ES stands for the percentage of chloroform (read on the scale AC), ET stands for the percentage of water (read on the scale CB), while ER stands for the percentage of acetic acid (read on the scale BA).



Geometry and Mixtures

It will be noticed that for point E, there is 30% water, 30% acetic acid, and 40% chloroform, the sum of which is 100%. For point R, there is 80% water, 10% acetic acid, and 10% chloroform, the sum of which is 100%. For point K, there is 30% water, 50% acetic acid, and 20% chloroform, the sum of which is again 100%.

The fact that such a diagram can be used to represent three-component systems is based on the plane geometry theorem stated as follows: In an equilateral triangle the sum of the perpendiculars to the sides from any point within the triangle is a constant. It can easily be shown that this constant is equal to any one of the three equal altitudes of the triangle. Move the interior point to any one of the vertices as a limit case. Then two of the perpendiculars reduce to zero, while the remaining one is the same as the altitude of the triangle. For a three-component percentage diagram, the altitudes are each divided into one hundred one-per cent intervals.

Where the three components are partially soluble in each other, the phase diagrams are useful in physical chemistry in studying varying mixtures with respect to saturation, homogeneity, and layer-forming. In metallurgy they are useful in developing alloys with particular properties, where the alloys are mixtures of three pure metals.

The applications described above are not too difficult for high school students, nor do their scientific content require much orientation. They are primarily mathematical in content, but they speak eloquently of the role mathematics plays in the sciences.

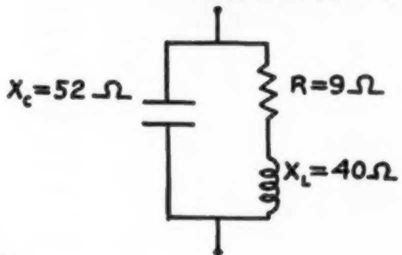
For some time now I have been examining a master's thesis entitled "Applications in Geometry for Instructional Purposes" written by William Carter in 1949 at Ohio State University (130 pp.). Mr. Carter discusses the theory and significance of applications in geometry and presents a number of well-chosen examples. I plan to point out some of those whose simplicity and pointedness might well cause them to be overlooked by the geometry teacher.

An Example of a "Complex Number"

The length of a bee stinger, alleged to be 12.03125 inches, is a "complex" number, since .03125 inches of it is "real," while the other 12 inches is purely "imaginary" when he stings you!

MULTIPLICATION and DIVISION of COMPLEX NUMBERS

PROBLEM: COMPUTE THE IMPEDANCE OF A CONDENSER IN PARALLEL WITH A COIL.



$$Z\ TOTAL = \frac{Z\ condenser \times Z\ coil}{Z\ condenser + Z\ coil}$$

$$Z\ CONDENSER = -j52 \text{ ohms}$$

$$Z\ COIL = 9 + j40 \text{ ohms}$$

$$Z\ TOTAL = \frac{-j52(9+j40)}{-j52+9+j40} = \frac{2080 - j468}{9 - j12}$$

$$= \frac{2080 - j468}{3(3-j4)} \times \frac{3+j4}{3+j4} = \frac{8112 + j6916}{75}$$

$$= \frac{2704}{25} + j\frac{6916}{75} = 108.16 + j92.21 \text{ ohms}$$

$$|Z\ TOTAL| = \frac{2132}{15} = 142.13 \text{ ohms}$$

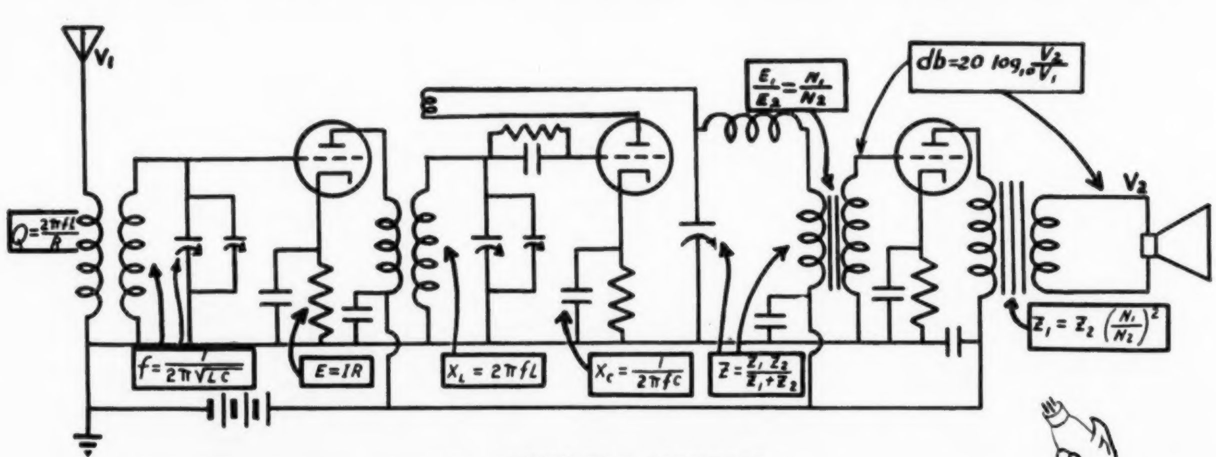
If we had wrongly used $\frac{|Z\ condenser| \times |Z\ coil|}{|Z\ condenser| + |Z\ coil|} = \frac{52 \times 41}{52 + 41}$ *we would have obtained* $\frac{2132}{93} = 22.91$ *ohms which is less than* $\frac{1}{4}$ *of the correct result.*

PREPARED IN COLLABORATION WITH THE MENS AND WOMENS MATHEMATICS CLUBS OF CHICAGO

PREPARED BY THE SIGNAL SCHOOLS 6TH SERVICE COMMAND

POSTER # 19

MATH IN A THREE TUBE RECEIVER



DEFINITIONS

Q = *Factor of merit of coil. The greater the Q the sharper the tuning.*

f = *Frequency allowed to pass (this part of radio selects station)*

$N_1(N_2)$ = *Number of turns on primary (secondary) coil.*

A = *Ratio of voltage given out to voltage received by the tube.*

$X_L(X_C)$ = *Impedance of coil (capacitor.)*

db = *Voltage gain decibels.*

THIS IS A DIAGRAM OF A THREE TUBE RECEIVER

SYMBOLS

Vacuum tube
Resistor
Coil
Capacitor
Battery

PREPARED IN COLLABORATION WITH
THE MEN'S AND THE WOMEN'S
MATHEMATICS CLUBS OF CHICAGO

PREPARED BY SIGNAL SCHOOLS
6TH SERVICE COMMAND

POSTER # 28

MEETINGS, INSTITUTES, WORKSHOPS

The third annual meeting of the **Michigan Council of Teachers of Mathematics** will be held **May 2–4, 1952** at St. Mary's Lake Camp, near Battle Creek. Teachers of mathematics at all grade levels from the elementary grades to the college will profit greatly from the stimulating professional activities which have been planned by the program committee. Participants will also find time to become well acquainted with their colleagues from other schools as well as to take advantage of the excellent recreational opportunities which this camp provides. Teachers from other states and Canada are also welcome to attend.

Registration will begin on Friday afternoon, May 2, at 1:00 and the opening general meeting of the conference will be held at 8:00 P.M. Plans will then be made for participation in the discussion groups to be held the following day. The evening will be concluded with a "get-acquainted hour" of games and conversation with coffee and snacks at 10:00. The first general meeting on Saturday morning will begin at 9:00 and will be followed by the meetings of the discussion groups. The afternoon and evening meetings include a third meeting for discussion groups, a general meeting and recreation periods. On Sunday morning, a half-hour devotional period will be followed by the last general meeting and the closing dinner.

A varied array of topics with competent leaders for the small discussion groups have been planned by the members of the program committee. Instruction and practice in constructing multi-sensory aids will be provided also.

Fifteen dollars ($15) will be the approximate total cost to each individual. This will include dormitory sleeping accommodations for Friday and Saturday nights and all meals beginning with dinner Friday evening and ending with dinner Sunday noon. This charge also includes the nightly coffee and snacks and the registration fee of one dollar ($1). Reservations should be made early since the camp can only accommodate about 150 persons. Each reservation must be accompanied by a two dollar ($2) deposit which will later be deducted from the total charge and should be sent to Miss Gertrude V. Pratt, Central Michigan College of Education, Mt. Pleasant, Michigan.

The Association of Mathematics Teachers of New York State will hold its **Second Annual Meeting** on **May 3, 1952.** Lt. Col. R. C. Yates of the U. S. Military Academy at West Point will speak at the general morning session on "The Mathematics Laboratory" and at the general sessions in the afternoon Dr. William Betz of Rochester Public Schools and Dean Joseph Seidlin of Alfred University will use as their respective topics, "Developing a Constructive Program for the Future" and "Ideal Preparation in Mathematics for College Entrance." Nine sectional meetings for teachers in colleges and in elementary, junior high and senior high schools have been planned for the morning and seven discussion groups will meet in the early afternoon. Copies of the program may be obtained by writing to Miss Elaine Rapp, High School, Oceanside, New York.

Indiana University will hold its **Fifth Annual Workshop for Teachers of Mathematics** on the campus at Bloomington, Indiana, **June 23** through **July 5.** Opportunities will be provided for teachers to work on their specific problems. There will be lectures from representatives of industry, as well as a mathematics laboratory in which the workshop membership may work on projects to their liking. Recent text materials, textbooks, audio-visual aids, and other items of interest to mathematics teachers will be displayed. For those who wish it, university credit will be allowed. A program will be available by the first part of May. If interested, write to Philip Peak, Workshop Director, Education Building, Indiana University, Bloomington, Indiana.

Louisiana State University will hold its **Third Annual Mathematics Institute** from **June 15–21.** Plans for the program include a geometry laboratory as well as discussion groups in algebra, geometry, arithmetic, junior high school mathematics, and enrichment materials. The discussions will be led by experts in these fields. In addition, there will be lectures given by outstanding people in mathematics and related fields. Excellent rooms and meals will be provided on the campus at reasonable rates. A June trip to interesting Louisiana can include a week for profit and enjoyment on the beautiful campus of Louisiana State University. Further information and a copy of the program may be obtained by writing to Dr. Houston T. Karnes, Director, Mathematics Institute, Louisiana State University, Baton Rouge, La.

The University of Houston, Houston, Texas, will hold its **Second Mathematics Institute, July 28–31, 1952.** This Institute is being sponsored by the College of Education, the Mathematics Department of the College of Arts and Sciences, and the College of Business Administration. Specialists in the field of mathematics teaching who will participate in the Institute program are: Dr. Edwina Deans, Elementary Supervisor, Arlington, Virginia; Dr. Herbert F. Spitzer, Principal of the University Elementary School and Associate Professor of Education, University of Iowa; and Dr. John R. Mayor, Chairman of the Department of Education and Professor of Mathematics, University of Wisconsin. Mr. Martin Wright, Associate Professor of Mathematics, University of Houston, will

(*Continued on page 309*)

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

Anyone who has a learning aid which he would like to share with fellow teachers is invited to send this department a description and drawing for publication. Or if that seems too time consuming, simply pack up the device and mail it. We will be glad to originate the necessary drawings and write an appropriate description. All devices submitted will be returned as soon as possible. Send all communications to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

THE SUM OF THE EXTERIOR ANGLES OF ANY POLYGON IS 360°

The device pictured in Figure 1 may be used to demonstrate this fact for the hexagon and octagon in a rather unique way. To construct the device procure two pieces of plywood or masonite each 12″ wide, 18″ long, and $\frac{1}{4}$″ thick. On one of the pieces draw a regular hexagon and a regular octagon with the exterior angles clearly defined. Open a compass to a radius of $1\frac{1}{2}$″ and with the vertices of the exterior angles as centers complete sectors of equal circles for each exterior angle of the two polygons. Thus all exterior angles in the two figures will be angles of sectors of equal circles. Next cut out the sectors on a jigsaw. In order to lift the sectors easily insert a small screw eye in each.

Cut a circle with a diameter of 3″ in the lower middle section of the board. Because the saw may have cut away part of the angles of the sectors it may be necessary to line the circle with a piece of metal such as an old clock spring. Of course the circle could also be cut slightly smaller than 3″, but the metal spring provides a tidy arrangement for holding the sectors in place firmly while at the same time allow-

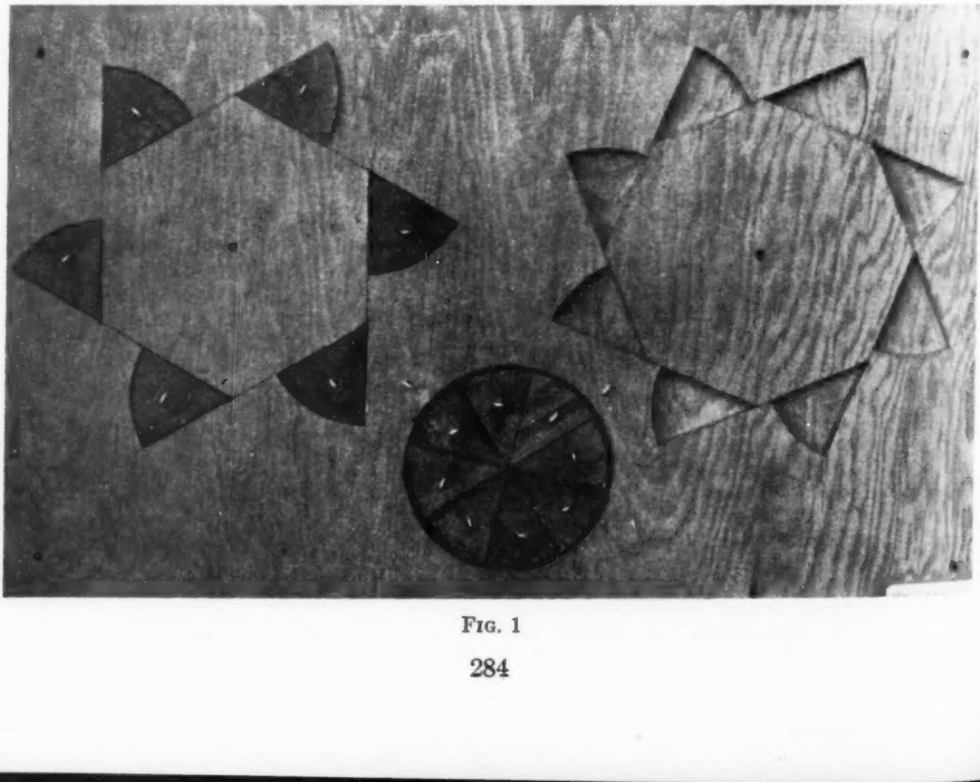


FIG. 1

ing them to be removed easily.

Fasten the piece of plywood with the sectors and circle cut out to the second piece with six screws or nails as indicated by the black dots in the picture. A coat of shellac will finish the work satisfactorily. Perhaps painting the sectors corresponding to the exterior angles of the hexagon and octagon different colors would make the device even more vivid. In using the device as a teaching aid it should certainly be permissible to refer to the sectors directly as the "exterior angles" since there is a one-to-one correspondence. The word "sector" was used in the description above simply as a convenience in describing the various phases of the construction.

The teaching aid described in this article was made by Mr. William Lawler, a student in the mathematics classes at Millersville State Teachers College, Millersville, Pennsylvania.

George R. Anderson
Millersville State Teachers College
Millersville, Pennsylvania

The Thousand Board

How many are one thousand? Children after they reach the stage of facile rote counting and some enumeration, love to talk glibly about large numbers and strive independently to extend their number horizons. A board 14″×20″×1″ covered with a sheet of aluminum 1/32″ thick in which 1,000 holes have been drilled is an effective device for helping children think about number and operations with numbers. In the design illustrated in Figure 2 there are 1,000 holes spaced $\frac{3}{8}''$ apart to form a rectangle having 25×40 holes. The holes are $\frac{1}{8}''$ in diameter and $\frac{5}{8}''$ deep. To use the device 1,000 nails or small pieces of wood of different colors which will fit into the holes are needed.

The thousand board is primarily designed to help children broaden their understanding of numbers after a foundation of counting has been developed. However, the board is also useful in assisting children to determine "how many." The child may enumerate groups placed by the teacher or produce groups as specified.

It is from the basic idea of 10 that the child may be led to discover that 10 groups of 10 form 100 and 10 hundreds constitute 1,000. Thus the board can be used to help children develop an understanding of decimal notation. Placing the nails in the holes helps give a clear understanding of 1,000.

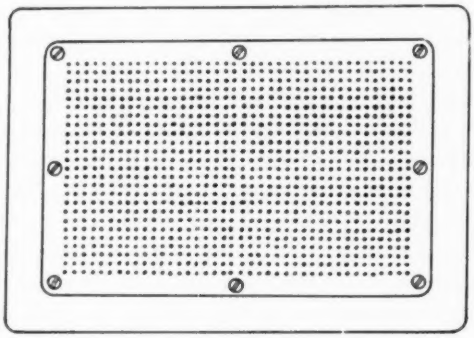


Fig. 2

The orderly arrangement of the thousand board makes the counting of large numbers certain and easy.

The meaning of addition as the regrouping of two or more groups into a single group and the meaning of subtraction as the division of a single group into two groups when the size of one group is known can be demonstrated with the board, or the child may discover these concepts himself by handling the board in accordance with instructions given by the teacher. In addition he will place nails to form groups to be combined, move them together to form one group, and count out the number resulting from the operation of regrouping. In subtraction he will form one group of nails, separate out a desired number, and count those which remain.

Multiplication may be introduced as successive addition. In multiplying 5 by 6, for example, form a row of 6 nails and designate it 1 by 6. Place a second row above the first and call it 2 by 6. Continue this process until we have 5 by 6. In this way the child may discover that multi-

plication is successive addition. By following this procedure the child can form the multiplication tables on the thousand board. The rectangular arrangements will show that the product of two numbers does not depend upon the order of multiplication. Thus the cumulative law of multiplication can be illustrated. Division, also a process of regrouping and often presented simultaneously with multiplication, can be demonstrated on the thousand board by placing a group of nails such as 30, and then successively removing 6 to find how many 6's are contained in 30.

The device is valuable in helping the teacher capitalize upon the child's natural curiosity about numbers and stimulating his thinking about numbers. Certain arrangements from the field of number theory are fruitful in extending the child's number understanding. The nature of odd and even numbers is an illustration. An even number is defined as one which is evenly divisible by 2, and any number which is not is odd. By using the thousand board one can show even numbers as two parallel rows of equal length. That such a group is divisible by 2 can be shown by placing a line between the two rows and calling attention to the fact that there are equal numbers on both sides. When the number is odd there will be 1 more in one row. It is only a matter of arrangement to illustrate the truth of the following theorems:

(1) The sum of any two even numbers is an even number.
(2) The sum of any two odd numbers is an even number.
(3) The difference of any two even numbers is an even number.
(4) The difference of any two odd numbers is an even number.
(5) Any even number can be written in the form $2M$
(6) And odd number can be written in the form $2M+1$ or $2M-1$.
(7) The sum of any even number and any odd number is an odd number.
(8) The difference of any even number and any odd number is an odd number.

Mathematicians give much attention to prime numbers—that is, integers which are not evenly divisible by any number except themselves or unity. According to this definition 2, 3, 5, 7, 11, 13, 17, and 19 are prime numbers. The child can soon learn that there is an essential difference between prime numbers and others by trying to place numbers representing these into rectangular arrangements. Is 29 a prime number? Give the child 29 nails and have him try to form a rectangle. Have him do this in an orderly manner by first placing 2, then successively 3, 4, 5, and 6 on a side. The child will soon see that it is useless to go beyond the 6 on a side. Since he has tried all possible combinations he will realize that 29 cannot be formed into a rectangle and is therefore prime.

The thousand board can also be used in pointing out to the child that two arbitrary collections, A and B, have only three possible aspects of size: (1) A equals B; (2) A is greater than B; or (3) A is less than B. The equality of A and B may be shown by placing nails or pegs in parallel rows to form a one-to-one correspondence. When A is greater than or less than B these facts can be readily recognized. With exercises of this kind the teacher can show the meanings of "equal," "greater than," "less than," "how many more," "how many less," and "enough to make."

All through arithmetic the concept of a many-to-one correspondence is encountered. Our system of notation is only one of its numerous applications. We are using this idea when we speak of miles per gallon, bushels per acre, miles per hour, etc. We can use one edge of the thousand board to represent number of children and the other edge to represent sheets of paper needed to make a booklet. It should be pointed out that we select holes in a straight line when we count A units over and B units up. Two classes of numbers may also be represented by two kinds of

objects such as nails and pegs.

Another interesting exercise is to have children construct the squares of numbers within the limits of the board. The term square is evidently based on the fact that the product of a number by itself can be arranged in the form of a square.

DONNA NORTON
Eugene Field School
Rock Island, Illinois

A TRISECTION DEVICE BASED ON THE INSTRUMENT OF PASCAL

Even though it has been known for some time that the general angle cannot be trisected with unmarked straightedge and compasses the problem continues to be a topic of considerable interest. Almost perennially methods and mechanical devices are added to the constantly growing collection of schemes for doing the job. That none of these obey the classical restrictions imposed upon straightedge and compass constructions seems only to add fervor to interest in the problem. The device suggested here is similar to the instrument of Pascal described by Robert C. Yates in the *Eighteenth Yearbook* of the National Council of Teachers of Mathematics, except that an additional slotted bar (AE) has been added.[1] (See Figure 3.) The device is also pictured in the *Seventeenth Yearbook*, but unfortunately this publication is out of print.[2] The proof is similar to that usually given for the

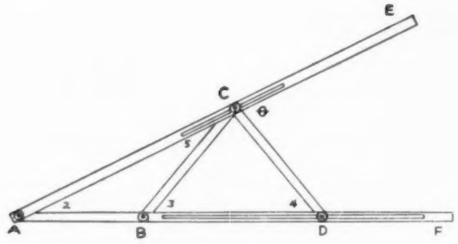


FIG. 3

marked straightedge construction of Archimedes.

To build the device select two sturdy strips of wood $\frac{5}{8}''$ wide and $27''$ long, and two others $\frac{5}{8}''$ wide and $8''$ long. Cut slots in the two long bars according to the following directions: In bar AE begin the slot $12''$ from A and cut it $6''$ long in the direction of E. In bar AF begin cutting the slot $9''$ from A and cut it $14\frac{1}{2}''$ long in the direction of F. Each slot must be wide enough to allow a $\frac{1}{8}''$ brass bolt to slide along its length. Holes large enough to accommodate $\frac{1}{8}''$ brass bolts should be drilled at points A, B, C, and D in the various bars as needed. Holes near the end of any

(*Continued on page 293*)

[1] Robert C. Yates, "Trisection," *Multi-Sensory Aids in the Teaching of Mathematics*, (18th Yearbook of the National Council of Teachers of Mathematics [New York: Bureau of Publications, Teachers College, Columbia University, 1945]), pp. 150–51.

[2] *A Source Book of Mathematical Application*, (17th Yearbook of the National Council of Teachers of Mathematics [New York: Bureau of Publications, Teachers College, Columbia University, 1942]), p. 212.

Proof

(1) $\angle 2+\angle 5=\angle 3$.	(1) An exterior angle of a triangle is equal to the sum of the two opposite interior angles.
(2) $\angle 2=\angle 5$.	(2) The angles opposite the equal sides of an isosceles triangle are equal.
(3) $2(\angle 2)=\angle 3$.	(3) Substitution axiom.
(4) $\angle 3=\angle 4$.	(4) Reason (2).
(5) $2(\angle 2)=\angle 4$.	(5) Substitution axiom.
(6) $\angle 2+\angle 4=\angle\theta$.	(6) Reason (1).
(7) $\angle 2+2(\angle 2)=\angle\theta$.	(7) Substitution axiom.
(8) $3(\angle 2)=\angle\theta$.	(8) Addition.
(9) $\angle 2=\angle\theta/3$.	(9) Division axiom.

AIDS TO TEACHING

Edited by

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School of Education
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Boston, Massachusetts

and

Donovan A. Johnson
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Minneapolis, Minnesota

BOOKLETS

B. 94—A Descriptive and Evaluative Bibliography of Mathematics Films

Available from Henry W. Syer, Boston University School of Education, Boston

Booklet; 96 pages; $8\frac{1}{2}'' \times 11''$; mimeographed; $.75

Description: According to the authors, Anthony B. DiLuna, Raymond F. Fleet, Jr., Milfred K. Hathaway, Jr., this publication is an attempt:

1. "To describe fully the mathematical content, the presentation of the mathematical content, and the physical aspects of each film;

2. "To evaluate each film as to technical make up, mathematical content, the presentation of the mathematical content, and educational purposes and uses;

3. "To give an over-all outlook of a large number of mathematics films with regard to the evaluative items in purpose two above."

To evaluate the films the authors constructed a check list which several mathematics teachers used to record their judgment of the film.

After a description of the method of evaluation, the next fifty-six pages are given over to a description of sixty-four films. For the convenience of the reader the description and evaluation of each film is classified under course headings. Each description includes the cost, source, length, and recommended use as well as the content of the film. The balance of the booklet summarizes the teacher responses on the evaluation check list. This summary indicates the rating the films received in several categories so that the teacher can quickly find out the general quality of a film as well as its most appropriate function in the classroom.

Appraisal: This bibliography not only provides the teacher with complete information as to the content, source, etc., of mathematics films, but will help her to be discriminating in her selection and use of films. It is unfortunate that the evaluation of each film is not given with the description, but the authors are to be commended for the comprehensive over-all evaluation of films and the recommendation for improvements in mathematics films based on an extensive evaluation study. More studies of this nature are needed if mathematics films are to find their proper place in our educational program. Every mathematics teacher should have this bibliography.

B. 99—Universal Bevel Protractors

Browne and Sharpe Manufacturing Co., Providence, R. I.

Booklet; 3 pages; $9'' \times 6''$; free

Description: The purpose of this brief booklet is to teach the reader how to use a universal bevel protractor. It discusses the principle of the vernier, the care and use of bevel protractors, and how to read the vernier scale on the bevel protractor. Two examples are given to illustrate the reading of the vernier.

Appraisal: This brief treatment of bevel protractors will be adequate for the teacher to learn how to read the vernier scale. However, the small print, the brief description, and the few examples will hardly be satisfactory for a novice.

B. 100—Micrometer Caliper

Browne and Sharpe Manufacturing Co.; Providence, R. I.

Folder; $5\frac{1}{4}'' \times 17''$; free

Description: The purpose of this folder is to teach the reader how to use a micrometer. It discusses the following topics: the principle of the micrometer caliper, the care and use of calipers, reading the micrometer, and adjusting the micrometer. It includes illustrative problems on reading the micrometer.

Appraisal: This is a very brief treatment of micrometers. It is written for the craftsman who is to use it and thus uses the vocabulary of the trade, such as "pitch of the screw." The few problems included are illustrated with drawings that clarify the explanation of each reading.

CHARTS

C. 36—Tree of Knowledge

Museum of Science and Industry, Jackson Park, Chicago 37, Ill.

Chart; $25'' \times 38''$; \$.25 plus \$.20 postage.

Description: This chart which was painted as a mural for the Chicago World's Fair by John Norton is a diagram, in the form of a stylized tree with the sciences of mathematics, zoology, botany, bacteriology, psychology, anthropology, physiology, astronomy, physics, chemistry, and geology as roots and the applied sciences of education, sociology, economics, medicine, pharmacology, public health and hygiene, forestry and agronomy, animal husbandry, marine and military engineering, aeronautic engineering, electrical engineering, mining engineering, architectural engineering, and mechanical engineering as branches.

Appraisal: Since mathematics appears as the most basic of the sciences at the bottom of the tree, this chart is particularly interesting to teachers of that subject. It is still a graphic and decorative way to relate our subject to other areas. Many would argue with the choice of subjects and the arrangement of their relationships, but no one would disagree with the idea that sciences need and use interrelationships.

EQUIPMENT

E. 72—Addi-Fax

E. 73—Multi-Fax

The Playway Games, C. N. McRae, Publisher, 18 Division St., Sidney, N. Y.

Games; $2'' \times 3\frac{1}{2}''$ cards; \$1.50 each

Description of E. 72: This game consists of 105 cards with a number printed on each and four value cards giving the answers to all number combinations of numbers below ten. The purpose of the game is to make as many combinations, called "Faxs" or "books," as possible. A Fax always contains three cards. Two of the cards are addends and the third is the sum. Thus, cards 9, 3 and 12 make a Fax. All two figure numbers are used as sums. Value cards are to be used by the players to find sums that they do not know. A variety of games are described to lend interest in the use of the cards. The cards are labeled I or II according to the difficulty of the combination, so that the game can be adapted to the level of the players.

Description of E. 73: This game is exactly like Addi-Fax except that a "Fax" consists of two factors and a product.

Appraisal of E. 72 and E. 73: The cards for these games are printed on good quality plain cardboard and the variety of rules suggested for the players make these games well worth the price. They will furnish the teacher with material for games and contests that should stimulate interest in practice and drill.

E. 74—Fracti-Fax

The Plaway Games, C. N. McRae, Publisher, Sidney, N. Y.

Game; 97 cards; $2'' \times 3\frac{1}{2}''$; $1.50

Description: This game consists of two sets of cards. Set I contains forty-five cards with a fraction printed on each card. The purpose of the game is to make as many books or "Faxs" as possible. A book or "Fax" consists of 5 cards all of which have equivalent fractions. For example, cards 1/2, 2/4, 3/6, 4/8, 5/10 make a "Fax." Set II contains fifty-two cards. Each card of this set contains either a fraction or a decimal or a per cent in large print and the two equivalents in small print. An odd card, 1/12 ($8\frac{1}{3}\%$) is marked "Duke." The rules for playing a variety of games are included.

Appraisal: The cards for this game are printed on good quality, plain cardboard. The variety of game rules, and the printing on uniform cards make this game well worth the price. Since the equivalent values are printed on the cards of set II very little thinking is required of the pupil to make "Faxs." These cards will furnish the teacher with material for games and contests that should stimulate interest in practice and drill.

E. 75—Num-Bo

The Plaway Games, C. N. McRae, Publisher, Sidney, N. Y.

Game; Number blocks and cardboard base; $3.00

Description: The board for these games is a rectangle with two rows of circles, nine circles in each row. The circles in the first row are numbered from 1 to 9, and those in the second row are blank. Twenty-four cylindrical wood blocks are numbered as follows: ten 1's; four 2's; three 3's; two 4's; and one each of 5, 6, 7, 8, 9. Each block from 1 to 9 increases in size by the thickness of the number "1" block.

Appraisal: This device can be used by pupils to discover a variety of number facts. One of the first may be to relate the number symbol to its numerical value in terms of the number of unit blocks needed to represent this number. This should give a "feeling" of the comparative size of the numbers less than ten. The instructions included with the device describe a game in which the pupils are to select unmarked blocks to represent any number called. The blocks can also be used to discover the number combinations for numbers less than ten.

E. 76—Wham

The Plaway Games, C. N. McRae, Publisher, Sidney, N. Y.

Game; $10'' \times 18''$ board; disks; value cards; $3.00

Description: Wham is a board game which uses a plywood board about 18″ by 10″, having a rim on all sides. A numbered section at one end is the court for shooting numbered disks. These disks are shot from the opposite end with small paddles. By using different rules for scoring, the number experiences may be varied to fit the level of the player or to give practice on varied types of arithmetic work. Value cards supply the facts needed to score games correctly.

Appraisal: Although manual skill and luck both enter into the playing of this game, it will furnish another means of building arithmetic skills while having fun. A teacher with some ingenuity will be able to adapt its pattern to a wide range of mathematical skills from elementary arithmetic to secondary school mathematics.

E. 77—Bobby Duck

The Plaway Games, C. N. McRae, Publisher, Sidney, N. Y.

Game; Plywood duck and wood weights; $11.00

Description: This device consists of a balancing plywood duck, twelve inches long, mounted on a wooden base and upright supports, fifteen wood weights and

ten colored plastic weights. Ten of the fifteen wood weights are numbered from 1 to 10, according to their weights, and five from 1 to 5. The ten plastic weights are each equal in weight to the unit wood weight. The duck will balance when the sums of the weights on the opposite end rods are equal. The black mark at the bottom of the duck will then be in line with the uprights, indicating that the answer is correct.

Appraisal: This device can be used for a variety of experiments and games to build an understanding of numbers and processes. The meaning of numbers can be discovered by balancing any number with unit weights. Using only the numbered weights the addition and subtraction facts can be discovered or demonstrated, for example, number 7 placed on one end rod can be balanced by the 4 and 3; by the 6 and 1; by the 5 and 2. Simple multiplication can be shown through the use of identical weights. Similarly multiple addition can be illustrated. Although the device is made for primary arithmetic it could be adapted to junior high school use to show the meaning and solution of algebraic fractions. Blocks of unknown weights could be used for variables and the balancing known weights as constants.

FILMS

F. 70—Search for Security

Institute of Life Insurance, 488 Madison Avenue, New York 22, N. Y.

B&W ($25.00); one reel; 17 min.

Description: This film presents the history and operation of insurance companies. After an introductory statement on the need for protection against risk, it pictures the beginning of mutual aid societies in England in 1690 to share the risk of marine shipping. The first American insurance organization was a Presbyterian clergy organization in Philadelphia in 1756. These early societies soon found the need for information regarding probability and the mortality table. The operation of insurance companies is described showing the distribution of funds collected by the companies in premiums. The film discusses the kinds of investments made by insurance companies and the different kinds of policies which can be purchased.

Appraisal: Except for the illustration of probability applied to the mortality table and the distribution of the premium dollar, this film contains no mathematics. Its primary object is to sell life insurance. In the process, however, it does give information on how a life insurance company operates. It could have been more interesting if it had included pictures of disasters, such as a fire, to show the need for insurance rather than having the actors discuss this need. The photography is fair. The commentary is clear, but the conversation of the actors is sometimes difficult to follow.

FILMSTRIPS

FS. 106—The Slide Rule: Proportion, Percentage, Squares and Square Roots

United World Films, Inc., 1445 Park Avenue, New York 29, N. Y.

B&W ($1.00); 44 frames.

Educational Details: U. S. Office of Education, Training film Division of Visual Aids, Federal Security Agency, FS 354. Frames 2 through 6 give a review of percentage and proportion. Frames 8 through 13 provide sample problems in proportion and percentage. Frames 15 through 29 provide a review of the A and D scales, squares and square roots. Frames 31 through 36 provide sample problems obtaining squares and square roots. Frames 38 through 40 provide a review of the procedure for solving one unknown side of a right angle triangle. Frames 42 through 44 provide sample problems in solving for the unknown side of a right angle triangle.

Description: There are illustrations of a slide rule, showing the C and D scales, and the A and B scales, and their relationship. Then other illustrations show the particular part of the slide rule used in a problem

on a magnified scale. The first 13 frames show simple examples of solving proportions and finding percents. Slightly more difficult problems in proportions and percentages are given in frames 7 through 14. Frames 15 through 36 give examples and illustrations involving squares and square roots, with the A, B and D scales; also using the C scale. In frames 38 through 44 the solution of a right triangle is shown, using the slide rule for finding squares and square roots: the Pythagorean Theorem.

Appraisal: I think this filmstrip does meet the objectives of the mathematics curriculum. The filmstrip should be used during the teaching of the use of the slide rule, to be followed immediately with problems worked on the slide rule. The ideas are not developed too fast. The amount of ground covered is just about right. The material is accurate and authentic. I think a film would be a better way to develop the ideas of slide rule use. The picture details are not too clear at times. (Reviewed by Orville A. George, Public Junior College, Mason City, Iowa)

FS. 107—Aerial Navigation: Radius of Action Returning to Same Base

Official filmstrip No. 1–67; United World Films, Inc., 1445 Park Ave., N. Y. 29 N. Y.

B&W ($0.72); 15 frames; 1942.

Educational Details: For senior high school and junior college students. The chief topics covered are: Dead Reckoning Problems: finding the radius of action; meaning of terms airspeed, fuel supply, wind vector, heading, course, groundspeed, altimeter, temperature gauge, true air speed; formulas for time out and radius of action.

Description: By means of diagrams, vectors are drawn showing the relationship of course out and course back, wind, and true airspeed. Computation of radius of action and time out are done by means of a scale drawing, using a compass.

Appraisal: This filmstrip attempts to do one thing well: that is to develop formulas for the radius of action and the time on the course out. At least not too much is attempted in a few frames. Perhaps it would be improved if a few simple examples were worked out as illustrations of the ideas developed. The development of ideas is not too fast. The lessons it teaches would be of chief value to a student pilot who plans to put the ideas into immediate practice. (Reviewed by Orville A. George, Public Junior College, Mason City, Iowa)

SOURCES OF MATERIAL FOR LABORATORY WORK

SL. 22—Angle Mirror

E. M. Stoddard, 47A Jones Street, Hingham, Mass.

Kit; Unassembled parts; $0.60

Description: This kit consists of two mirrors, two wooden blocks for mounting the mirrors, one fiberboard base, one spool to be used as a handle, bolts and nails. The only additional material needed to make the angle mirror is a hammer and glue. The kit includes a sheet of directions for assembling the model. These directions also discuss the use of the angle mirror and the optical principle on which it is based.

Appraisal: This is an inexpensive kit that can be used by students to build a very useful angle mirror. It is very simple to assemble the model. All parts are cut and holes for bolts and nails have been drilled. All one has to do to assemble the kit is to glue the mirrors to blocks of wood, bolt the blocks to the base, and nail the spool to the base. The entire operation need not take more than fifteen–twenty minutes. This angle mirror can then be used for field work to lay out right triangles, to lay out a circular course, or to measure inaccessible distances.

SL. 23—Sextant

E. M. Stoddard, 47A Jones Street, Hingham, Mass.

Kit; Unassembled parts; $1.25

Description: This kit consists of a pro-

tractor, handle block, $5\frac{3}{4}$-inch cross bar, a spool for a sighting tube, index arm, two mirrors, blocks for mounting the mirrors, plastic screens, screws and nails. The only materials needed to assemble the parts are hammer, screw driver, glue, scissors, and a little steel wool. The kit includes a sheet of directions for assembling. These directions also discuss the use of the sextant and the optical principle on which it is based.

Appraisal: This kit can be quickly and easily assembled by students. It is inexpensive and yet accurate enough for student use. Since all parts are completely cut and holes for nails and screws drilled, it is simple to assemble. Mathematics teachers can no longer omit field work from their courses because of the lack of equipment. This kit will fill a long felt need for practical, inexpensive student-built instruments.

SL. 24—Transit

E. M. Stoddard, 47A Jones Street, Hingham, Mass.

Kit; Unassembled parts; Wood; $1.25

Description: This kit consists of a $4\frac{1}{2}$ by 5 inch base block, a bevelled pivot block, an 8-inch sighting bar, two uprights, four protractors, lead shot plumb bob, screws and nails. The additional materials needed to assemble the kit include a hammer, compass, scissors, ruler, pliers, scotch tape, glue and fine thread. The kit includes a sheet of directions for assembling and using the model.

Appraisal: This kit will make it possible for every mathematics teacher to have a number of simple transits available for field work. The cost is ridiculously low, the task of assembling extremely simple, and the instrument sufficiently accurate for student projects in indirect measurement. This is an excellent buy.

SL. 25—Plastic Cube with Inscribed Octahedron

E. M. Stoddard, 47A Jones Street, Hingham, Mass.

Kit; Plastic; Unassembled; $1.25

Description: This kit contains the six pieces of plastic needed to form a two-inch cube. The kit also includes a length of red thread, two scrap pieces of plastic, and round toothpicks. The additional materials needed to assemble the kit include glue (plastic), a wooden forming cube, knife, flat toothpicks, needle, and razor blade. The kit includes complete directions on how to assemble the cube.

Appraisal: The pieces of plastic are cut accurately so that a careful assembly job will result in an attractive model. In assembling the model it is necessary that the directions be followed very carefully to avoid marring the plastic. The assembling of this model will give experience and ideas that can be used in the building of a variety of plastic models.

Note: On any order for SL. 22–SL. 25 of $5.00 or over, a 10% discount is allowed.

Devices

(*Continued from page 287*)

bar should be located $\frac{1}{4}''$ from the tip. In drilling the holes care must be taken that $AB=CD=BC$ from center to center. Assemble the device as illustrated with $\frac{1}{8}''$ brass bolts, washers and nuts.

To trisect a given angle θ place C on the vertex of the given angle, fix CE along one side of θ, and slide the bolt at D along the slot in AF until CD coincides with the second side of θ; then angle 2 will be equal to $\theta/3$.

An examination of the proof on page 287 will show that the use of this particular device will provide opportunity for application of two important theorems from plane geometry.

THE MATHEMATICS LABORATORY
Monroe High School

MATHEMATICAL RECREATIONS

Edited by Aaron Bakst
135-12 77th Avenue, Flushing 67, N. Y.

This department adheres to the view that mathematical recreations are not activities which are used in some haphazard manner. Unfortunately for the profession of the teaching of mathematics the methodology of this subject ignores mathematical recreations. In some quarters there still persists the belief that humor and laughter are injurious to learning processes.

In the preceding issues of The Mathematics Teacher several typical examples of mathematical recreations were presented and the thesis proposed that a recreational problem may be "created" by the teacher. It was also suggested that recreational problems need not be confined to arithmetic situations exclusively. This thesis and this suggestion will now be extended so as to include the *integration* techniques in the teaching of mathematics. In other words, recreational problems may be so constructed that the pupils may be called upon to utilize all the skills and all the meanings which were developed on the different levels of mathematical instruction.

This department offers an observation that certain phases of mathematical instruction suffer from the lack of the development of effective meaning. Specific criticism is leveled in the direction of the teaching of the coordinate geometry methods in elementary algebra. It is doubtful whether the teaching of graphs and of the solution of equations by means of graphs contains any seeds which, when planted, will sprout in effective meanings. The abstract and the theoretical treatment of this phase of algebra is weak from the point of view of attaining any definite degree of understanding of the powers of the methods of coordinate geometry. One may ask the question: "The pupils have learned how to graph and how to solve simultaneous equations graphically. So what?" What would be the honest answer of a teacher to this question?

The best test of the understanding of the methods of coordinate geometry may be obtained in terms of their applications to situations which ordinarily may not attract our attention. Such situations would be challenging to both the teacher and the pupil. But once the pupil grasps the general idea of this method, he will (at least, this is to be hoped for and, if feasible, cultivated) employ this method in other problem situations.

The illustrative problem which is presented here belongs to the level of difficulty of ninth year algebra. True, and this is not to be hidden from the teacher, this department suggests that the simple ideas of probability be presented to ninth year pupils. However, this problem of teaching the simple ideas of probability to these pupils should not offer any difficulties (unless certain localities may object to the teaching of this subject on moral or other reasons). It is very doubtful that present time pupils have not come across the statements of "odds" and other concepts of probability. Newspapers and other media of communications are replete with references to probability (and not necessarily gambling). Let us be realistic about this topic. Modern industry, science, warfare, and so on depend very much on the applications of probability.

Mr. Jones has a date with Miss Richards. This is a theater date. The curtain is scheduled to go up at 8:15 p.m. They agreed to meet at 7:45 p.m. Mr. Jones

was there on time. But Miss Richards, exercising the prerogatives of a lady, did not show up on time. After waiting 30 minutes, Mr. Jones left her ticket at the box office (this was their agreement), and went inside alone. What is the probability that they would have met within the first 30 minutes after 7:45 P.M.? Originally, he agreed to wait an hour.

The problem seems to be very difficult. Or is it?

On the coordinate system XOY (see Figure 1), which is scaled for minutes on both axes, let us denote Mr. Jones' arrival by the symbol x, and the arrival of Miss Richards by the symbol y. In order that they meet it is necessary that

$$|x-y| \leqq 30.$$

We construct on the coordinate plane a square whose sides are 60. The area within the square contains all the points (which are expressed in terms of the coordinates of x and y) which indicate all the possible time combinations within the specified hour for which the meeting was agreed. On the other hand, the area of the square whose sides are 30 represents the possible combinations for meeting within the first 30 minutes.

The area of the square whose side is 60 is 3,600. The area of the square whose side is 30 is 900. The probability that there would be a meeting within the first 30 minutes is then

$$\frac{900}{3600}=\frac{1}{4}.$$

In other words, the odds are 3 to 1 in favor that Miss Richards would be late.

This problem may be varied. Suppose we ask "What would be the probability of their meeting (Mr. Jones left after 30 minutes' waiting) if their respective arrivals were unspecified within the period between 7:45 and 8:45 and the times of their arrivals were independent of one another?" In other words, Mr. Jones could be late also.

This situation is illustrated in Figure 2. The shaded portion inside the square, whose sides are 60, indicates the possible time of their mutual meeting. The probability is then

$$\frac{60^2-30^2}{60^2}=\frac{2700}{3600}=\frac{3}{4}.$$

Thus, in order to "keep harmony" it would have been better if Mr. Jones were late too. The odds are now 3 to 1 in favor that he will meet Miss Richards.

If the readers of THE MATHEMATICS TEACHER have any comments regarding the above observations as well as some interesting problems of similar nature, this department will be glad to receive them.

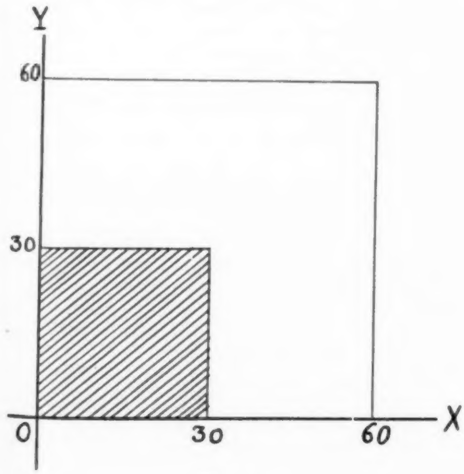

FIG. 1

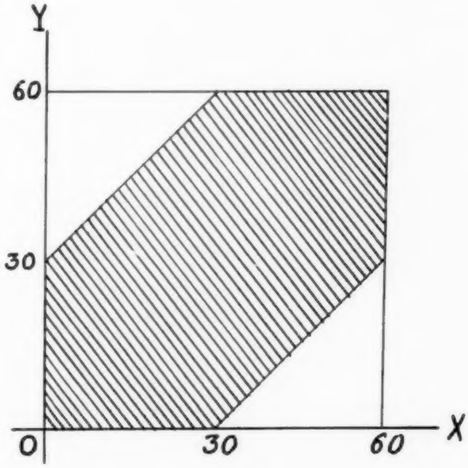

FIG. 2

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Edited by William L. Schaaf
Department of Education, Brooklyn College, Brooklyn, N. Y.

Emerging Practices in Mathematical Education

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Correction: In the bibliography for "Psychology of Learning Mathematics" in the January 1952 issue, page 61, under heading 4, "Learning Process", the statement that Mr. J. Allen's article appeared in *School* (*Secondary Edition*) should be deleted.

BOOK SECTION

Edited by Joseph Stipanowich
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

Junior High School

Mathematics in Action, Book 1 (Third ed.), by Walter W. Hart and Lora D. Jahn. Cloth, xi+324 pages, 1952. D. C. Heath and Co., 285 Columbus Ave., Boston 16, Mass. $2.12.

Mathematics in Action, Book 2 (Third ed.), by Walter W. Hart and Lora D. Jahn. Cloth, vii+294 pages, 1952. D. C. Heath and Co., 285 Columbus Ave., Boston 16, Mass. $2.24.

Mathematics in Action, Book 3 (Third ed.) by Walter W. Hart and Lora D. Jahn. Cloth, viii+354 pages, 1952. D. C. Heath and Co., 285 Columbus Ave., Boston 16, Mass. $2.40.

High School

Algebra for Problem Solving, Book One, by Julius Freilich, Simon L. Berman, and Elsie Parker Johnson. Cloth, 568 pages, 1952. Houghton Mifflin Co., 2 Park St., Boston, Mass.

Second Algebra (Second ed.), by Virgil S. Mallory, State Teachers College, Montclair, New Jersey; and Kenneth C. Skeen, Union High School and Junior College, Taft, California. Cloth, vii+480 pages, 1952. Benjamin H. Sanborn and Company, 221 East 20th Street, Chicago 16, Illinois. $2.48.

Solid Geometry, by Walter W. Hart, formerly of the University of Wisconsin, and Veryl Schult, Supervisor of Mathematics, Washington, D. C. Cloth, ix+198 pages, 1952. D. C. Heath and Co., 285 Columbus Ave., Boston 16, Mass. $2.40.

College

Analytic Geometry, by W. K. Morrill, The Johns Hopkins University. Cloth, xii+383 pages, 1951. International Textbook Co., Scranton 9, Pa. $3.50.

Business Mathematics, by Walter F. Cassidy and C. Carl Robusto, St. John's University. Cloth, vii+304+109 pages, 1952. Prentice-Hall, Inc., 70 Fifth Ave., New York 11, N. Y.

Trigonometry, Plane and Spherical, by Lloyd L. Smail, Lehigh University. Cloth, xii+406 pages, 1952. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, N. Y. $3.75.

Basic Mathematics for Technical Courses, (Second ed.), by Clarence E. Tuites, Rochester Institute of Technology. Cloth, x+438 pages, 1952. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. $3.75.

Applied Mathematics for Technical Students, (Rev. ed.), by Murlan S. Corrington. Cloth, xiii+273+134 pages, 1952. Harper and Bros., 49 East 33rd St., New York 16, N. Y. $4.00.

The Design and Analysis of Experiments, by Oscar Kempthorne, Iowa State College. Cloth, xix+631 pages, 1952. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. $8.50.

Electromagnetics, by Robert M. Whitmer, Rensselaer Polytechnic Institute. Cloth, ix+270 pages, 1952. Prentice-Hall, Inc., 70 Fifth Ave., New York 11, N. Y. $5.00.

Meaningful Mathematics, by H. S. Kaltenborn, Memphis State College. Cloth, xiv+397 pages, 1951. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N.Y. $4.75.

Teaching Mathematics in the Secondary School, by Lucien Blair Kinney, Stanford University; and C. Richard Purdy, San Jose State College. Cloth, xvi+381 pages, 1952. Rinehart and Co., 232 Madison Ave., New York 16, N. Y. $5.00.

Miscellaneous

General Education in Science, edited by I. Bernard Cohen and Fletcher G. Watson. Cloth, xviii+217 pages, 1952. Harvard University Press, Cambridge, Massachusetts. $4.00.

Mathematics, Its Magic and Mastery (Second ed.), by Aaron Bakst. Cloth, xiv+790 pages, 1952. D. Van Nostrand Co., 250 Fourth Ave., New York 3, N. Y. $6.00.

Chemical Calculations, An Introduction to the Use of Mathematics in Chemistry, by Sidney W. Benson, University of Southern California. Cloth, xi+217 pages, 1952. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16 N. Y. $2.95.

Matter and Motion by J. Clerk Maxwell. Paper, 163 pages, Dover Publications, 1780 Broadway, New York 19, N. Y. $1.25. (Cloth edition, $2.50.)

REVIEWS

Arithmetic 5: The World of Numbers, Dale Carpenter, Edith M. Sauer, G. Lester Anderson. New York, The Macmillan Co., 1950. iii+316 pp., $1.68.

Arithmetic 5: The World of Numbers is one of a series of six books, grades three through eight. This book has qualities that should immediately capture and hold the interest of the learner. The materials are pleasingly arranged, the pictures are not only colorful but functional, and the print is clear and of good size. The authors have shown unusual skill in suiting the problem situations to the daily contacts and interests of fifth graders.

The orderly, progressive organization of materials, emphasizes the fact that arithmetic is a system of related ideas. Throughout the book emphasis is placed on the interrelationship of processes and on the use of numbers. Much care is taken to provide materials to help the learner build new meanings on old ones, and to build understanding before practice. A variety of tests at the end of each chapter provides excellent opportunity for self-evaluation and for diagnosis of weaknesses.

The program for fifth grade includes a reteaching section of the four fundamentals, addition and subtraction of fractions, common measures, and an introduction to decimal fractions. It is quite possible that many groups may not be able to adequately carry such an inclusive program.

This series gives promise of creating greater interest in the study of arithmetic and in turn improved skill in use of numbers.—OLIVE G. WEAR, Fort Wayne, Indiana.

Intermediate Algebra, Paul K. Rees and Fred W. Sparks. New York, McGraw-Hill Book Company, 1951. viii+328 pp., $3.25.

This text is designed for the college student who has had only one year of high-school algebra. The first nine chapters cover the usual topics through simultaneous quadratic equations; these are followed by four chapters on ratio, proportion, and variation; logarithms; the progressions; and the binomial theorem. The authors have pitched their discussions to the level of the average student and have strived to use a clear, informal style. A great number of illustrative examples are solved step by step in considerable detail. Since drill work is essential in a course on this level, an abundance of carefully graded problems are provided with answers given to all even-numbered problems and to one-half of the odd-numbered problems. This feature was particularly attractive to the reviewer who was sufficiently impressed by the text to adopt it for use this year. The book meets our own needs very well.—H. D. LARSEN, Albion College, Albion, Michigan.

Machine Shop Mathematics, Second Edition, Aaron Axelrod. New York, McGraw-Hill Book Company, 1951. xi+359 pp., $3.60.

This text in applied mathematics is designed for the student of a vocational or technical high school who has had "at least one or two or more years of high school mathematics and science." The chapters on the measuring tools and equipment of the machine shop are fully presented, replete with clearly stated problems, copiously illustrated with up-to-date photographs and excellent engineering drawings.

The chapters on shop trigonometry, geometric constructions, and weights and measures are impressive presentations of applied mathematics for the high school. This second edition contains three new chapters, "Shop Arithmetic," "Percentage and Its Applications," and "Ratio and Proportion." The 64-page appendix is composed of twelve tables, including sine bar constants, areas and volumes, and 5-place trigonometric functions.

Other than the designed purpose of this text, it should make a valuable addition to the library of every high school mathematics classroom.—R. W. MOLLENDORF, McKinley High School, Chicago, Illinois.

Self-Help Algebra Workbook, L. S. Walker and and George E. Hawkins. Chicago, Scott, Foresman and Co., 1950. 79 pp., $.80.

The general purpose of the Self-Help Algebra Workbook is to help the student gain a more thorough and complete mastery of algebra and a greater understanding of the basic algebraic ideas. The authors try to accomplish this purpose by giving two kinds of work: (1) carefully planned drills or tests to show the pupil exactly what he knows and what he has forgotten, and (2) study units which discuss the harder topics in the drills and give more practice on the important ideas.

There are thirty-three standardized drills. The first two and the last review the important topics in arithmetic. The remaining thirty provide for about one drill a week. A self-help progress chart is provided and space is allowed for listing the pages in the textbook where explanations are found on how to do the examples which the pupil had worked incorrectly. After every six drills there is a study lesson to review the more difficult examples.

There is a determined and detailed thoroughness evident in the planning for drill, checking, relearning from the text, and retesting—DOROTHY S. WHEELER, Bulkeley High School, Hartford, Conn.

Elements of Algebra, Lyman C. Peck. New York, McGraw-Hill Book Company, Inc., 1950. xiii+230 pp., $2.75.

Elements of Algebra has been written "primarily for college students who have had no algebra in high school and thus presupposes no mathematical background other than ordinary arithmetic."

The book presents a logical development of the laws of the number system, progressing from definitions and operations with positive integers through addition, subtraction, and multiplication with signed numbers, to the use of these learnings in "Equalities." (Chapter 5). Division is presented as an outgrowth of multiplication, and fractions as a "natural result of division of integers." Factoring, Graphic Methods, and Systems of Linear Equations are included to assist in solving equations. The chapter on quadratic equations presents square root, Pythagorean Theorem, and a section on imaginary numbers. This is followed by a chapter on complex numbers, and the book is concluded with a 13-page chapter of problems from chemistry and physics.

Although this book covers (for the most

part) the material of the traditional first-year algebra text, yet the treatment is from a more mature point of view than that usually commended by the ordinary high school freshman. Teachers of high school algebra would do well to examine this text for "inspiration" for their brighter pupils.—ELINOR FLAGG, Illinois State Normal University, Normal, Illinois.

Mary Everest Boole, A Memoir with some Letters, E. M. Cobham. Essex, England, The C. W. Daniel Company Limited, 1951. Available through the Concord Book Shop, Inc., 36 South Michigan Ave., Chicago 3, Illinois. xiv+131 pp., $2.00.

The mathematics teacher who worked out an extremely valuable theory of "the relation between the conscious and the unconscious" before Freud's work was available in English, here lives as a human being. This biography assists our thinking the more surely because of that humanness.

The little volume not only gives us the external facts of Mrs. Boole's life as her own "Collected Works" can hardly do, but is a tribute to the love and affection which Mrs. Boole inspired in her pupils. The author was one of these pupils.

Mrs. Boole is remembered by many of us for her: "The Preparation of the Child for Science," "Philosophy and Fun of Algebra," and her introduction to "A Rhythmic Approach to Mathematics."—LAURA E. CHRISTMAN, Senn High School, Chicago, Illinois.

The Elements of Mathematical Logic, Paul Rosenbloom. New York, Dover Publications, Inc., 1950. iv+224 pp., $2.95.

On the dust jacket of this book one finds the statement "The mature mathematician will find this an excellent introduction to mathematical logic." The reviewer felt that this statement was precisely correct. The book seems too difficult for all but those with a rather mature mathematical background. If, however, the reader has had previous acquaintance with the subject the degree of mathematical maturity need not be so great.

As the quoted statement implies, the book is intended for mathematicians; logicians will not quite agree with Professor Rosenbloom's presentation is as they would have it. Perhaps this is necessary though, since the author is aware of the fact that the logicians' presentations usually fail to arouse the interest of the mathematician.

The treatment of the subject is thorough in the sense that most of the major problems are discussed and all of the more important approaches to the subject are mentioned. Excellent bibliographical remarks are to be found at the end of the book.

Professor Rosenbloom's style of presentation is quite individualistic; the reader should enjoy many humorous and salty comments which one finds interspersed throughout. There seem to be several errors in the text which were not caught in proof reading. These may cause some confusion at points. On the whole the reviewer thought the book accomplishes its purpose and fills a heretofore rather large gap in the available presentations of mathematical logic.—D. H. POTTS, La Mesa, California.

Operational Calculus Based on the Two-Sided Laplace Transform, Balth. van der Pol and H. Bremmer. New York, Cambridge University Press, 1950. xiii+415 pp. $10.00.

The material in this book is based on the two-sided Laplace transform defined by

$$f(p)=p\int_{-\infty}^{\infty}e^{-pt}h(t)dt$$

instead of the more familiar form

$$f(p)=\int_{0}^{\infty}e^{-pt}h(t)dt.$$

By defining the Laplace integral as $f(p)/p$ instead of as $f(p)$, the unit function in the t-domain corresponds to $f(p)=1$. By allowing the lower limit to be $-\infty$ instead of 0, the operational rules are simplified, and a much larger class of functions can be used; furthermore, the ordinary operational caculus is included as a special case.

Chapter I contains an extensive history of the operational concept, and throws light on the Heaviside controversy. The importance of the Fourier integral as basis of operational calculus is stressed in Chapter II. In succeeding chapters the operational rules are carefully developed, and copious examples are provided. An extensive treatment of the unit function and the impulse function is provided in Chapter V, including discussion of the sifting property of the Dirac δ function. In the following two chapters the question of convergence and series expansion are discussed.

The remaining twelve chapters are devoted to application of this foregoing theory to solution of linear differential equations with constant coefficients; simultaneous linear differential equations with constant coefficients; linear differential equations with variable coefficients; step functions and other discontinuous functions; difference equations; integral equations; partial differential equations; and simultaneous operational calculus. A "grammar" of operational rules, and a "dictionary" of operational relations provided in the last two chapters afford a ready reference as well as an index to the contents of the book itself.

This is a truly scholarly work; the examples used range from problems arising in electrical engineering to those encountered in the most advanced branches of mathematics. One feature of this work is the application of operational calculus to the theory of numbers, a field which is by no means exhausted. This book will be useful to the engineer as well as to the research worker in pure mathematics.—E. W. BANHAGEL, Detroit, Michigan.

The Kernal Function and Conformal Mapping, Stefan Bergman. New York, American Mathematical Society, 1950. viii+161 pp. $4.00.

This volume is number 5 in the Mathematical Surveys series. As the author states in his preface, its purpose is "to present a number of methods and principles which are of wide applicability in such branches of analysis as function theory, partial differential equations, differential geometry, etc." Needless to say, the level of presentation is fairly advanced; in fact the reader should already be acquainted with the above-mentioned subjects and others of advanced level such as Lebesgue integration.

The principal tool is the introduction of the kernal function, which is characteristic of a given domain in the complex plane. This function is defined in terms of a complete system of orthonormal functions in the domain. The fact that the kernal function can be expressed in terms of a complete orthonormal system makes possible the numerical solution of many boundary-value problems and hence the applicability of the technique to problems in applied mathematics.—D. H. POTTS, La Mesa, California.

Anthology in Educology, Lowry W. Harding. Dubuque, Iowa, Wm. C. Brown Company, 1951. xv+78 pp., paper covers, $1.50.

Teachers take themselves too seriously, and teaching has become an increasingly grim business badly in need of the saving grace of humor. With this as his premise, Lowry W. Harding of the Department of Education at Ohio State University, coining a word "educology," creates an "Association for Preservation of Humor in Educological Workers." As "curator" of this organization he presents this "report," a collection of specimens of educological humor, most of them light verse. They deal with teachers, parents, students, and educational trends.

The selections have been gathered from various sources; five are from *The Saturday Evening Post*. Of the fifty items included, some six or eight deal directly or indirectly with mathematics. One by the editor of this collection bears the title "Mathematical Rigor Mortis." Another, "The Tables are Bare" appeared originally in THE MATHEMATICS TEACHER. The Editor proposes to add to his collection and issue further "reports" from time to time.—KATHARINE E. O'BRIEN, Deering High School, Portland, Maine.

"**If we could canvass the leading thinkers of the Western world today, asking them to name those fields in which our civilization has attained preeminence, I venture the guess that they might place mathematics first among our sciences and music first among our arts.** Both these disciplines have contributed something cardinal and distinctive to the fabric of our culture. It would be logical to expect that history teachers, explaining the genesis and evolution of the contemporary mind would deem modern music and mathematics worthy of special attention as perhaps the most original creations of Western Man. Yet how seldom this is done. . . .

" '**Mathematics may well claim to be the most original creation of the human spirit not only in the field of science but in any field whatever.**' (Preserved Smith. *A History of Modern Culture*. New York: Harper Brothers, 1930. Volume I, p. 89.) The scientific wonders of our age were spelled out in mathematical formulas before they became transforming facts. It would be difficult to overemphasize the effect of mathematical science in shaping those instruments of power and precision that distinguish our civilization most decisively from any other civilization that mankind has known. Yet how many general history texts can you cite that discuss the development of mathematics with the attention reserved for the revival of Latin literature in the Renaissance or the poetry of the Romantic Revolt?

"Such discrimination is dangerous. The historian can no longer afford to treat mathematics as the Cinderella in our house of culture, not now that she is married to Science, the fairy prince. There are too many members of our society, charming people many of them, who admit gaily that they cannot balance their check stubs against a bank statement and require help to add a bridge score. Of course the historian is not to blame because we have so many mathematical illiterates in our midst. It is not his fault if we think it a higher proof of culture to quote Dante in the original than to place a decimal point correctly. But in a society governed by statistics, it might be better if our history texts paid more honor where honor is due, and mentioned Laplace or Lobachevski with the same respect accorded to Petrarch or Milton. The mathematician is the Merlin of this modern age; the apparitions invoked by his genius are all about us; our skies are darkened with his runic spells. Yet if all other sources perished, what testimony to his genius would be preserved in our general history texts? Histories of science aside, I can think of only one recent tribute to the mathematicians that raises them to the elevation they merit. On the famous list of one hundred great books compiled at St. John's College, the mathematicians outnumber the poets almost two to one, and the works on science bulk larger than all the works cited from imaginative literature."—Geoffrey Brunn, "World History: The Problem of Content," *Social Education*, XVI (Jan. 1952) 3–7. (Excerpt reprinted by permission from The National Council for the Social Studies and the author.)

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by John R. Mayor *and* John A. Brown
The University of Wisconsin, Madison, Wisconsin

Mathematics Conference for High School Students

Approximately eighty students from eight high schools attended Eastern State High School's second annual mathematics conference for high school students on Monday afternoon and evening, November 19, 1951 at Eastern Illinois State College, Charleston, Illinois.

Mathematics students of Eastern State began to meet and greet their guests at 3:45 p.m. By 4:15 all had been registered, pinned with name cards, and shown to the meeting place for the first session.

The program for the afternoon consisted of favorite problems presented by individual students. Mr. Berlen Flake, mathematics teacher from Cumberland Community Unit Schools, presided as chairman. Mr. Lawrence Ringenberg, head of the college mathematics department, and Mr. David J. Davis of the college department were special guests.

Don Hopkins, Casey High School, explained what magic squares are and showed how to construct them. John Willingham, Eastern State presented a puzzle problem: "If a person drives from here to St. Louis (150 miles) at an average rate of 30 m.p.h., how fast must he return to average 60 m.p.h. for the trip?" Marilyn Lawrence, Neoga High School, showed us how to write any rational number using only integers and fractions with numerators of one. Orville Bradford, Marshall High School, set up a plane table and showed how to use it to measure distances indirectly. Wanda Tipsword, Cumberland, presented a number puzzle. Jim Farr, Neoga High School, presented one of those problems about bananas, banana thieves, and a monkey; he had the answer, but wanted to know how to get it. (Some of the teachers present worked so hard on this one that they lost out on the rest of the afternoon program.) Jim Bruce and Richard Phipps of Eastern presented a problem they had brought up in their junior algebra class: They had solved a set of three linear equations for x, y, and z in terms of a constant, a. Since a part of the solution was $y=a$, they thought they ought to be able to replace the constant, a, with y in the original equations, solve the new set of equations, and come out with a definite solution. They had wanted to know why this "wouldn't work." They presented the explanation at which their class had finally arrived. At this point in the proceedings, the favorite problem discussion was broken off so that the treasure hunt could start.

A mathematical treasure hunt, designed by Mr. Harris and a committee of student teachers, closed the afternoon program. Twelve teams participated in the treasure hunt. They had been drawn so that each team had at least one member from the host school and no two members from any other one school. The treasure hunt required a series of measurements with protractors and meter sticks.

The group met for dinner in the college cafeteria. Pat Price, president of the Eastern State High School student council, served as master of ceremonies and introduced the groups from the visiting schools. Dinner favors were programs containing some logic problems and a pin puzzle.

After dinner everyone hurried back to

the Main Building to finish the treasure hunt. Prizes for the treasure hunt were presented at the evening session. The program continued with presentation of favorite problems by James Adams, Eastern; Armand Loffredo, Casey; John Hutchings, Casey; and Mary Lea Nelson, Eastern.

The high school students present were unanimously in favor of another such conference next year. Requested suggestions for improvement brought forth the proposal that each school submit some problems to be mimeographed and distributed a few weeks before the conference, so that everybody could have a chance to try each problem before hearing somebody else give the solution.

Students and teachers were present from high schools at Arthur, Casey, Charleston, Cumberland, Eastern State, Effingham, Marshall, and Neoga. Host teachers for the conference were Mr. Raymond P. Harris and Miss Gertrude Hendrix.

GERTRUDE HENDRIX
Eastern Illinois State College
Charleston, Illinois

A PROJECT IN INDIRECT MEASUREMENT

We have in my class room a few simple instruments for measuring—an angle mirror or two, a hypsometer, a couple of sexttants, two home-made transits and two or three other instruments which pupils have made. The pupils of my plane geometry classes become somewhat acquainted with these instruments as occasion arises throughout the school year.

In the spring, after we have studied similar triangles and a bit of trigonometry, on a day when classes and weather seem ready for it, I invite all pupils who have the same vacant period I do to go with me that hour to a park across the street, where I give them a chance to handle and to use these instruments. Each student present selects a group of pupils within his class whom he would like to lead and to instruct the next day. These group leaders next decide what heights and distances in the park they would like to measure and what instrument they would prefer to use in each case. They explain to me and to each other the procedure they will follow.

The next day, weather permitting, the classes go to the park, each group within the several classes accompanying its leader. I move from group to group as leaders explain and pupils individually follow their guidance in making measurements. As soon as a group has used one instrument it exchanges instrument and position with another group. Thus during the hour every pupil has a chance to use several instruments, a secretary for each group jotting down the angles used, the measurements made. There is no confusion. Every student is busy the entire period; every student enjoys the hour.

The assignment is to determine each height and distance for which measurements were taken. The following day comparisons are made in class, discrepancies noted and discussed. Students after this experience and discussion may take the instruments from the class room during their vacant hours, after school or for week-ends and work with them on individual or group projects, presenting the results of their work in attractive posters which are displayed in the class room.

CLARICE KAASA
Red Wing High School
Red Wing, Minnesota

AN EXPERIMENT IN THE ACCELERATION OF SEVENTH GRADE MATHEMATICS PUPILS

Our school in Owatonna is set up, as most schools are, to meet the needs of the average child. The problem we have in seventh grade mathematics is the wide range of individual differences. The background of some of the pupils is very good while that of others is poor. The ability of some to grasp the material is high while that of others is very low. Pupils with low ability and poor background take more time and effort on the part of the instructor.

In order to meet these problems more efficiently we resorted to individualized instruction, whereby the bright student could progress as fast as his ability permitted and the slower pupil would go at a rate of speed according to his ability and could get more individual help from his instructor. We have found that by challenging the bright student and putting him on a competitive basis with his class by keeping new material before him so that the work isn't boresome, a greater interest is created in his work. The average child will try to keep up with the class; his progress isn't too difficult. But some of these pupils who are industrious and have a competitive spirit will extend themselves to try to keep up with the better students. The slow pupil who has a hard time with mathematics and gets discouraged easily is put on material which he can do and gradually works up. This type of person is our greatest problem because he is unhappy in his work when he has to compete with the best students. We have found that when we give him work that he can master and he sees he is learning something in the class, he becomes more industrious and is happy about it.

How do we go about to accomplish all this? As we stated before individualized instruction is the method. The materials used are (a) a workbook which has drills and problem sets; (b) the Strathmore Plan, which is a comprehensive set of exercises designed to cover specifically each and all of the mathematical ideas taught in arithmetic; and (c) a textbook used for reference. We spend the first four or five weeks of school in reviewing and preparing the class for their first drill in the workbook. When the review is completed, each student works individually on workbook drills. As soon as a student has corrected all errors and drilled more on the units that he made mistakes on, he takes a problems set based on the drill he has just completed. When he has completed the problems satisfactorily, he is ready to proceed to his second drill. The student who has fewer mistakes will progress more rapidly than the others. Consequently, he will finish the seventh grade work before the end of the year and will continue into the eighth grade work.

The result of this procedure is that we have a number of students each year who have not only completed seventh grade mathematics but have completed the eighth grade work as well. To put these students in the eighth grade would be mere repetition. The challenge to the pupil would be lost, and we felt that disciplinary problems would develop. What did we do about it? We are experimenting with a limited number of select pupils who have been accelerated from seventh grade mathematics directly into algebra with the ninth grade pupils.

The criteria used for selection were (a) scholarship, (b) test results, and (c) maturity.

(a) If the child had marks which were in the upper quartile of the seventh grade and if he had finished the eighth grade work by the end of the seventh grade, he would be eligible for consideration for advancement to ninth grade algebra.

(b) In order to be considered, a pupil must also rank high in three different tests:

(1) *Eighth Grade Achievement Test.*—This test is a standard test used in our school. Each child takes this test with the eighth grade and must rank in the upper quartile to be considered.

(2) *Iowa Algebra Aptitude Test.*—This test forecasts the ability of the student to work algebra. A pupil must rank above the 50th percentile to be considered.

(3) *General Intelligence Test.*—This test measures the general ability of the pupil to do school work. A pupil must rank well above average to be considered.

The first year we accelerated seven pupils to the algebra class. The scores these pupils earned on the various tests and the marks they have earned in algebra and geometry with the advanced class are

	A	B	C	D	E	F	G	Class Average
1. Eighth Grade Achievement Test Scores—administered at end of seventh grade	83	93	93	93	76	83	88	72
2. Algebra Aptitude Test Score—administered at end of seventh grade	89	83	91	94	73	79	87	54
3. Final mark in Algebra course taken with ninth grade	B	B	A	A	B	B	A	C
4. Grade Points out of possible 60 earned during fourth quarter of Algebra course	53	53	59	57	53	50	58	32
5. Final mark in Plane Geometry course taken with tenth grade	B	A	A	A	B	C	A	C

indicated in the table above. Pupils are designated by letters rather than by names. These people did not skip eighth grade work. They completed it at the end of the seventh grade as shown by the test results on line 1. The results at the end of one year of algebra reported on line 3 were uniformly good. We also found that these same pupils with one exception, did exceptionally good work in geometry as indicated on line 5 of the table.

This acceleration of these pupils will give them an opportunity to complete all of our mathematics program and at the same time strengthen their background in either science or language because they will have room for one more elective in their schedule. The pupils that were not accelerated also gained from our program as it results in smaller eighth grade classes. The teacher will be able to give more individual attention when the classes are smaller. This system has been working well so far in our school. The children like it and so do the teachers. If the program continues as it has and if the results are as satisfying, we expect to adopt the practice as a standard procedure.

Reported in the *Minnesota Mathematics Newsletter*, November 1951, by RAY H. STOCKTON, Owatonna High School, Owatonna, Minnesota.

HONOR WORK IN SECONDARY SCHOOL MATHEMATICS

Honor pupils are selected according to several criteria. The criteria are: (a) Intelligence Quotient (which is usually not less than 130); (b) Arithmetic records in the eight-year elementary schools; (c) Reading ability (obtained in accordance with special tests administered throughout the city school system); (d) General achievement (as demonstrated by the grade marks); (e) Personality.

Most of the information is secured from the pupils' Elementary School Record Cards which travel with the pupils as they progress from grade to grade.

If the pupils are selected for the special classes, they usually travel as a special Core Group in our school. In other words, they take English, Civics, and Science as a core course. If the pupils select Latin or Hebrew, they are not placed in the special mathematics classes. This is an administrative procedure because they cannot be programmed for the special mathematics courses.

Once the pupils (I refer to the ninth graders) become accustomed to their work they are permitted to engage in more abstract approaches to mathematics. They are permitted to develop the subject along more general lines. The computational phases of mathematics are not neglected, but the pupils are assisted in developing a sense for mathematics as a logical system. They are never told any rules, nor are they encouraged to use any rules unless they can discover them for themselves. In general, the pupils are guided so that the mathematics course is developed by themselves, and the teacher acts as a mediator. It does not take very much time before the pupils develop a certain amount of independence. On many

occasions they travel faster than is expected of them.

Very early in the term these pupils are encouraged to select some special books which may be in the recreational field or they select some popular work in the field of mathematics. The school library as well as the departmental library have collections of these books. These books are read without the assistance of the teacher. But, the pupils are encouraged to consult with the teacher whenever they find some difficulty which requires clarification. Generally, the pupils do not complete the reading of the entire book. But, as the result of this individual work, every pupil selects some special topic which interests him. He prepares a written report which is read by him to the class. The general reaction of the class is really remarkable. The pupils not only listen attentively, but at the end of a report a lively discussion takes up the remainder of the class period. It is impossible for me to list all the topics which are thus reported by the pupils. Generally, such topics may cover anything from navigation to some elementary steps in calculus.

By and large, the pupils develop a very keen interest in mathematics for the sake of mathematics. They are keen to learn the whys of mathematics. They are very much interested in the possibilities of selecting mathematics as a profession. At least once or twice a semester the teacher is compelled to spend the lesson periods for outlining to the pupils the scope of mathematics as well as the uses of mathematics in various professions. Some of the pupils come from families whose parents are professionals: engineers, mathematics and science teachers, school principals, doctors, and so on. These pupils contribute their own impressions to these discussions.

Generally, the work with bright pupils is not very easy. A teacher must be very alert, must have a mine of information handy and ready for use. The problem of homework assignment is a taxing one. The pupils usually demand more work than one would be expected to assign to them. They are very keen on the application of mathematics, and they dig out information which they bring to the class. The activity method of teaching prevails in these special classes. But, however lively the pupils may be, there is never a problem of discipline present with such classes.

Eloise B. Bakst
Jamaica High School
New York, N. Y.

A Faculty Looks at Mathematics for General Education

The success of a three quarter-hour course in mathematics for general education was evident to the faculty of Florida State University when the Faculty Senate increased the time given to the mathematics course in the general education program. In a Faculty Senate meeting, early in the spring of 1950, the motion to increase the time allocated to this work was made by a member who is not from the mathematics or science staff. This motion was passed without a dissenting vote and the class-room time was increased from about thirty class hours to approximately forty-eight. The three quarter-hour course was made a three semester-hour course, or the equivalent of a four and one half quarter-hour course.

This work is given by the mathematics department and is required of all students in every college of the University except in the College of Music. Any student in any college who demonstrates by examination that he has acquired mathematical knowledge sufficient for his needs is exempt from the university mathematics requirement. Approximately eight per cent of the entering student body of nineteen hundred and fifty-one were exempted from the course.

The purpose of this paper is to report the reaction of the faculty to the course and to review the reasons why the Faculty

Senate felt justified in giving more time to the study of mathematics.

When the Faculty Senate voted to increase the time devoted to the classroom study of this required basic course its members recognized that the student needs an understanding of mathematical language to make progress in many fields of study. Not only will the student use the language in his class work but he will meet situations in school and out of school when knowledge of mathematics will be very useful, perhaps essential.

In learning mathematics as a language the student acquires a tool and a language for quantitative thinking. Mathematics is the medium through which scientific argument is pursued. The natural scientists measure, describe, analyze, and explain. All this is possible through the instruments of formal logic and mathematics.

Use of mathematics is not confined to the natural scientists. The psychologist finds it a powerful tool in his work. The historians, the economists, and other social scientists, as well as people in other professions and trades, rely on mathematics as a language for expressing many fundamental ideas.

Man needs to know some basic materials in mathematics no matter what his vocation. Each student should learn how mathematics influences society and how it may affect his personal life. This knowledge may help him to determine the extent to which he should develop mathematics in his own life.

The theme of the basic course in mathematics for general education at Florida State University is, "To most people mathematics is primarily a language for expressing certain kinds of ideas." In selecting materials for the course an attempt is made to select problems that meet the needs common to students and adults. The course is functional. In it the student studies problems from the point of view of an adult and is expected to learn to make use of his mathematics in day to day living.

Emphasis is placed on the language of mathematics. Ratio and proportion are used extensively and developed more than in many basic courses. The topic of variation is studied and illustrated. Equations are taught as algebraic sentences and algebra is shown to be a more powerful tool than arithmetic. The student learns to translate arithmetic questions into equations and to solve the equations. Trigonometry is developed only as an extension of similar figures and indirect measurement. The closing lessons are used to orient the student into a world in which every one uses mathematics. The student is given the opportunity to learn the influence mathematics has had in the development of a civilization.

Many students enter the class with a "mental block" against mathematics. One of the first tasks of the teacher of the course is to find ways of removing the fears. The emphasis on mathematics as a language serves to improve the attitude toward the subject. Instructors often see a change in attitude when mathematics is referred to as a language instead of a science. The students seem to appreciate any approach that is somewhat different from that which they have experienced in their earlier studies of the subject. In this University the attitude toward mathematics seems to be improving and the faculty hopes this better attitude will spread into the elementary and secondary school communities of the State of Florida. When this develops, more effective teaching at those levels is to be expected. This will enable the general education mathematics staff in the University to pitch the basic course at a higher level.

The Faculty Senate expressed its approval of this basic course for general education by voting to devote more time to the subject. Departments in the University have approved of the work being done by making this course a prerequisite for certain departmental courses. During the last year the nursing school, the chemistry department, and the physics department

have made basic mathematics a requirement for specific courses in those fields. Individual teachers in many departments have been interested in the preparation of material for the course and have offered many suggestions that have been incorporated into the work. The course is looked upon as one that serves the needs of the student and of the University. To some it is a service course and for that reason alone many faculty members have expressed approval of the Senate action. The course appears to have a bright future on the Florida State University campus.

Hugh H. Hyman
Florida State University
Tallahassee, Florida

Meetings, Institutes, Workshops

(*Continued from page 283*)

address one of the general sessions. A mathematics laboratory for teachers of junior and senior high school mathematics will be directed by Miss Ida May Bernhard, Supervisor, College Laboratory School, Southwest Texas State Teachers College, San Marcos, Texas. All meetings will be held in air-conditioned rooms; residence can be secured in air-conditioned dormitories. For further information, write Miss Joyce Benbrook, Box 554, University of Houston, Houston 4, Texas.

Money-Managing Workshops for 160 educators will be held at four universities this summer in a program **to encourage better teaching of family financial security** in the nation's high schools and colleges under the sponsorship of the **Committee on Family Financial Security Education,** 488 Madison Avenue, New York 22, N. Y., of which Dr. Herold C. Hunt, general superintendent of schools in Chicago, is chairman. Dates for the workshops are as follows: University of Pennsylvania, Philadelphia, June 30 to August 9; University of Wisconsin, Madison, June 30 to August 22; Southern Methodist University, Dallas, Texas, July 14 to August 8; and University of Connecticut, Storrs, August 4 to August 22.

An outgrowth of two successful workshops sponsored by the Committee at the University of Pennsylvania in 1950 and 1951, which have already trained some 75 teachers, the present expanded program will make possible the training of some 160 additional teachers, principals, curriculum directors, supervisors and faculty members of teacher training institutions. One of the purposes of the program will be to develop materials which these and other teachers throughout the United States can use to teach their students the fundamental principles of managing their incomes and saving money. Scholarships which are to be awarded students attending the workshops have been made possible through grants to the universities by the Institute of Life Insurance. Subjects to be covered will include sources of income, budgeting, buying and borrowing on credit, home rental and ownership, life insurance, general insurance, investments, Social Security, and all other phases of financial planning. The teacher-students in the courses will also work in curriculum laboratories, developing units of teaching and class-room materials based upon the needs in their own schools and communities.

The "parent" workshop at the University of Pennsylvania will again award scholarships on a national basis which will include first-class rail transportation to and from Philadelphia and tuition charges. Participants will pay for their own board and room. Dean E. Duncan Grizzell of the University's School of Education is chairman of the University's Workshop Committee.

At the University of Wisconsin, six semester-hours credit and scholarships of $100 each (sufficient to cover the cost of tuition and rooms) will be awarded to those attending the workshop. Members will pay for their meals. Participants will be drawn chiefly from the North Central States. Applications should be addressed to Dr. Russell Hosler of the University's School of Education.

The workshop at Southern Methodist University will serve chiefly the Southwest United States. Four semester-hours will be awarded for successful completion of the workshop and scholarships will include room and board for the duration of the course. Participants will pay a tuition fee of $40. Dr. R. C. Watts is coordinator of this workshop.

In charge of the new workshop at the University of Connecticut, which will serve the New England states will be Dr. P. Roy Brammell, Dean of the School of Education. Participants will pay a tuition fee of $40 for four semester-hours of credit. Scholarships will meet the costs of board and room for the students during their stay at Storrs.

Faculty at all four workshops will be selected from staff members of the sponsoring universities' schools of education and business administration. Outside lecturers will also attend, drawn from local financial institutions. Although the workshops will not be exactly alike in program, all will be designed to promote more effective teaching of family financial security education in America's high schools. Students at the workshops will thus include teachers and others concerned with high school courses in social studies, business education, mathematics, family life education, homemaking, guidance, and related fields.

Program

Twelfth Summer Meeting

The National Council of Teachers of Mathematics

and the

Fourth New England Institute for Teachers of Mathematics

sponsored by

The Association of Teachers of Mathematics in New England
The Phillips Exeter Academy, Exeter, New Hampshire
August 21-28, 1952

THURSDAY, AUGUST 21

1:00–5:00 P.M. **Registration.** Academy Building

6:30 P.M. **Opening Banquet**

Topic: *Education in the Obvious*

Speaker: BANCROFT H. BROWN, Head, Department of Mathematics and Astronomy, Dartmouth College, Hanover, New Hampshire.

FRIDAY, AUGUST 22

9:00–10:00 A.M. **Lecture**

Topic: *The Meaning of Meaning*

Speaker: ERNEST NAGEL, Professor of Philosophy, Columbia University, New York, New York.

10:30–11:45 A.M. **Study Groups**

	Topic	Leader*
A1.	*Solid Geometry in the Curriculum*	J. Sutherland Frame
A2.	*Professionalized Subject Matter for Secondary School Teachers: Special Reference to Algebra*	Howard F. Fehr
A3.	*Senior High School Laboratory*	Allene Archer, Christina S. Little
A4.	*Effective Speaking in the Classroom*	Arthur W. Sager
A5.	*The Use of History in the Teaching of Mathematics*	Vera Sanford
A6.	*Puzzles and Recreations*	William R. Ransom
A7.	*How I Teach Long Division*	John R. Clark
A8.	*Informal Geometry in the Junior High School*	Mary C. Rogers
A9.	*The Training of Teachers; the Contribution of Formal College Courses*	Ralph Beatley, Virgil S. Mallory, Harold P. Fawcett, John G. Read

* "Who's Who" at the end of this program identifies the leaders of the study groups.

1:30–2:45 P.M. **Study Groups**

Topic	Leader
B1. *Professionalized Subject Matter for Secondary School Teachers: Special Reference to Geometry*	Bruce E. Meserve
B2. *How the Student's Classroom Participation Contributes to His Understanding and Growth in Mathematics*.	Martha Hildebrandt
B3. *The Use of Applications from Science in the Teaching of Mathematics*.	Henry W. Syer, Richard F. Brinckerhoff
B4. *The Technique of Explanation*.	Edmund C. Berkeley
B5. *Junior High School Laboratory*.	Gladys Schuder, Eleanor E. Taylor
B6. *"Teaching for Meaning" in Geometry*.	Maurice L. Hartung
B7. *How I Teach Division of Fractions*.	Ann C. Peters
B8. *How I Teach the Mechanics of Algebra*.	Charles H. Mergendahl
B9. *The Special Problems of the Small High School*. . .	Walter H. Carnahan

3:15–4:30 P.M. **Study Groups**

Topic	Leader
C1. *Studies in Geometrical Drawing and Linear Perspective*.	H. von Baravalle
C2. *Swap Shop: How do YOU do It?*.	George T. Major
C3. *The Assembling and Use of Prepared Kits. (Special Laboratory)*.	Elwood M. Stoddard
C4. *The Influence of Group Dynamics on Teaching and Learning*.	Hans Spiegel
C5. *The Junior High School*.	Veryl Schult
C6. *"Teaching for Meaning" in Algebra*.	John J. Kinsella
C7. *Number Readiness*.	Robert L. Morton
C8. *New Ventures in College Entrance Testing*.	William C. Fels
C9. *The Training of Teachers: the Contribution of Laboratory Experiences*.	Elizabeth V. Foster, Vincent J. Glennon, Mildred Stone, James H. Zant

8:15–9:15 P.M. **Lecture**

Topic: *The Industrial Engineer and Mathematics*

Speaker: LILLIAN M. GILBRETH, Industrial Engineer, Montclair, New Jersey.

SATURDAY, AUGUST 23

9:00–10:00 A.M. **Lecture**

Topic: *The Scientific Study of Language*

Speaker: EDMUND C. BERKELEY, Consultant and Actuary; President, E. C. Berkeley Associates.

10:30–11:45 A.M. **Study Groups**

Topic	Leader
A1, 2, 3, 4, 5 (Repeated on Saturday, Monday, and Wednesday. These are the Institute Groups.)	

A6. *Puzzles and Recreations*	William R. Ransom
A7. *How I Teach the Decimal Number System*	Harold E. Moser
A8. *Informal Geometry in the Junior High School*	Mary C. Rogers
A9. *The Training of Teachers. The Contribution of In-Service Training*	M. H. Ahrendt, John R Mayor, Rolland R. Smith, Henry W. Syer

1:30–2:45 P.M. **Study Groups**

Topic	Leader
B1, 2, 3, 4, 5 (Repeated on Saturday, Monday and Wednesday. These are the Institute Groups.)	
B6. *Using the Overhead Projector in Teaching Geometry*	Ralph F. Ward
B7. *"Teaching for Meaning" in Arithmetic*	Henry Van Engen
B8. *How I Teach the Mechanics of Algebra*	Charles H. Mergendahl
B9. *The Special Problems of the Small High School*	Walter H. Carnahan

3:15–4:30 P.M. **Study Groups**

Topic	Leader
C1, 2, 3, 4, 5 (Repeated on Saturday, Monday and Wednesday. These are the Institute Groups.)	
C6. *How I Teach Verbal Problems in Algebra*	Evan A. Nason
C7. *How I Teach Problem Solving in Arithmetic*	Robert L. Burch
C8. *New Ventures in College Entrance Testing*	William C. Fels
C9. *The Training of Teachers: Summary: What Makes a Teacher?*	

8:15–9:15 P.M. **Lecture**

Topic: *The Ubiquitous Golden Section*

Speaker: WAYNE DANCER, Chairman, Mathematics Department, University of Toledo, Toledo, Ohio.

SUNDAY, AUGUST 24

Church, recreation, mountain trips, beach trips, etc.

MONDAY, AUGUST 25

9:00–10:00 A.M. **Lecture**

Topic: *Mathematics in Operations Research*

Speaker: GEORGE P. WADSWORTH, Associate Professor of Mathematics, Massachusetts Institute of Technology; Consultant, Arthur D. Little, Inc., Cambridge, Massachusetts.

Study Groups 1 through 5 at each period are repeated. At 3:15 P.M. there will be a sixth study group, for elementary school teachers discussing *Teaching and Learning the Basic Concepts* and led by Ann C. Peters.

8:15–9:15 P.M. **Lecture**

Topic: *The Dynamic Beauty of Geometrical Forms* (Slides and demonstrations)

Speaker: H. VON BARAVALLE, Professor of Mathematics, Adelphi College, Garden City, New York.

TUESDAY, AUGUST 26

9:00–11:30 A.M. **Symposium:** *The Art of Teaching*

LEWIS PERRY, Principal Emeritus of Phillips Exeter Academy, Exeter, N. H., Chairman

JOSEPH T. WALKER, JR., Partner, Hornblower and Weeks, Boston, Mass.

J. SEELYE BIXLER, President, Colby College, Waterville, Maine.

A. ELIZABETH CHASE, Docent of the Yale Art Gallery, Yale University, New Haven, Conn.

LEONARD CARMICHAEL, President, Tufts College, Medford, Mass.

1:30–6:00 P.M. **Recreation** (Beach picnic, trips, etc.)

8:15–9:15 P.M. **Lecture**

Topic: *T. C. Mits Looks at Three Hundred Years of Mathematics*

Speaker: LILLIAN R. LIEBER, Head, Department of Mathematics, Long Island University, Brooklyn, New York.

WEDNESDAY, AUGUST 27

9:00–10:00 A.M. **Lecture**

Topic: *The Nature of Transmutation and Nuclear Fission*

Speaker: JOHN A. TIMM, Director, The School of Science; and Chairman, The Chemistry Department, Simmons College, Boston, Massachusetts

Study Groups 1 through 5 at each period are repeated. At 3:15 P.M. there will be a sixth study group for elementary school teachers, discussing *Teaching and Learning the Basic Concepts* and led by Ann C. Peters.

6:30 P.M. **Final Banquet**

Speaker: JAMES R. KILLIAN JR., President, Massachusetts Institute of Technology, Cambridge, Massachusetts.

ANNOUNCEMENTS

Notes About the Program

Your attention is invited to the fact that

1. The lecture on "the Meaning of Meaning" and the three study groups entitled "Teaching for Meaning . . . " constitute a connected series.
2. The four meetings on "The Training of Teachers" constitute a connected series.
3. The second day of two-day study groups is not a repetition of the first day but a continuation of the first day.
4. A room will be set aside for special showings of films and film strips.
5. A variety of teaching aids will be exhibited and an opportunity to make many of these models will be offered during the laboratory periods Those planning to join one of the laboratory groups should bring their own scissors.

Exhibits

Extensive exhibits of work by teachers and students are being planned. Publishers' exhibits will include the latest textbooks, professional books, aids to teaching, and also all books written by persons on this program.

Transportation

Exeter is conveniently reached by automobile (5 miles from U. S. Route 1), by train (Boston and Maine Railroad), by plane (to Manchester, N. H., or to Boston), or the Interstate Bus from Boston will leave you right at the school.

Mail

Mail should be addressed in care of

The Institute for Teachers of Mathematics
The Phillips Exeter Academy, Exeter, New Hampshire

Recreation

Tea will be served in The Big Room at 4:30 on Friday, Saturday, Monday, and Tuesday. "I'll meet you at The Grill" each night from 9:15 to 11:00. Tennis courts, swimming pool, and golf course are available. The ocean beaches are seven miles from Exeter. A trip to the White Mountains by bus is being planned for Sunday; a beach picnic for Tuesday. There will be other trips to places of historic interest in the vicinity. Families are invited to all social affairs, trips, and to use the recreational facilities. Children can be accommodated comfortably at the banquets because the speaking program will be held in a building different from the dining halls.

Who's Who Among the Leaders of the Study Groups

Ahrendt, M. H., Executive Secretary, N.C.T.M., Washington, D. C.
Archer, Allene, Instr. in Math., Thomas Jefferson High School, Richmond, Va.
von Baravalle, H., Professor of Mathematics, Adelphi College, Garden City, N. Y.
Beatley, Ralph, Assoc. Prof. of Educ., Harvard University, Cambridge, Mass.
Berkeley, Edmund C., Consultant and Actuary; President, E. C. Berkeley Associates.
Brinckerhoff, Richard F., Instr. in Sc., Phillips Exeter Academy, Exeter, N. H.
Burch, Robert L., Editorial Department, Ginn and Co., Boston, Mass.
Carnahan, Walter H., Consultant in Math. Educ., Purdue Univ., Lafayette, Ind.
Clark, John R., Prof. of Educ., Teachers College, Columbia Univ., New York, N. Y.
Fawcett, Harold P., Chairman, Dept. of Educ., Ohio State Univ., Columbus, O.
Fehr, Howard F., Head, Dept. of Math., Tchrs. Col., Columbia Univ., New York, N. Y.
Fels, William C., Secretary, College Entrance Examination Board, New York, N. Y.
Foster, Elizabeth V., Dir. of Training, State Tchrs. Col., Worcester, Mass.
Frame, J. Sutherland, Head, Dept. of Math., Mich. State Col., East Lansing, Mich.
Glennon, Vincent J., Assoc. Prof. of Educ., Syracuse Univ., Syracuse, N. Y.
Hartung, Maurice L., Assoc. Prof. of Math. Educ., Univ. of Chicago, Chicago, Ill.
Hildebrandt, Martha, Head, Dept. of Math., Proviso Twp. H. S., Maywood, Ill.
Kinsella, John J., Prof. of Educ., New York Univ., New York, N. Y.
Little, Christina S., Acting Head of Math. Dept., Girls' H. S., Boston, Mass.
Major, George T., Instr. in Math., The Phillips Exeter Academy, Exeter, N. H.
Mallory, Virgil S., Head, Dept. of Math., State Tchrs. Col., Montclair, N. J.
Mayor, John R., Chairman, Dept. of Educ., Univ. of Wis., Madison, Wis.
Mergendahl, Charles H., Head, Math. Dept., Newton H. S., Col., Newtonville, Mass.
Meserve, Bruce E., Assoc. Prof. of Math., Univ. of Illinois, Urbana, Ill.
Morton, Robert L., Professor of Education, Ohio University, Athens, O.
Moser, Harold E., Head, Dept. of Math., State Tchrs. Col., Towson, Md.
Nason, Evan A., Instructor in Mathematics, Phillips Academy, Andover, Mass.
Peters, Ann C., Professor of Education, Keene Teachers College, Keene, N. H.
Ransom, William R., Prof. Em. of Math., Tufts College, Medford, Mass.
Read, John G., Professor of Education, Boston University, Boston, Mass.

Rogers, Mary C., Head, Math. Dept., Roosevelt Jr. H. S., Westfield, N. J.
Sager, Arthur W., Instr. in Speaking, Gov. Dummer Academy, South Byfield, Mass.
Sanford, Vera, Professor of Mathematics, State Teachers College, Oneonta, N. Y.
Schuder, Gladys, Instr. in Math., Lane High School, Charlottesville, Va.
Schult, Veryl, Director of Mathematics, City Schools, Washington, D. C.
Smith, Rolland R., Coordinator of Mathematics, Public Schools, Springfield, Mass.
Spiegel, Hans, Director, International Student Center, Cambridge, Mass.
Stoddard, Elwood M., Instructor in Mathematics, Lincoln School, Hingham, Mass.
Stone, Mildred, Professor of Mathematics, Salem Teachers College, Salem, Mass.
Syer, Henry W., Associate Professor of Education, Boston Univ., Boston, Mass.
Taylor, Eleanor E., Teacher of Math., Central Jr. H. S., Quincy, Mass.
Van Engen, Henry, Head, Dept. of Math., Iowa State Teachers Col., Cedar Falls, Ia.
Ward, Ralph F., Director of Mathematics, Public Schools, Brookline, Mass.
Zant, James H., Prof. of Math., Oklahoma A. and M. Col., Stillwater, Okla.

Committee Chairmen

General chairman and Program chairman	Jackson B. Adkins, The Phillips Exeter Academy, Box 49, Exeter, N. H.
Publicity	Ruth B. Eddy, 666 Angell St., Providence 6, R. I.
Exhibits	Barbara B. Betts, D. C. Heath and Co., Boston, Massachusetts.
Recreation	Janet Height, High School, Wakefield, Massachusetts.
Housing	H. Gray Funkhouser, Cilley Hall, Exeter, N. H.
Laboratories	Christina S. Little, Girls' High School, Boston, Mass.
Representative for the National Council	Henry W. Syer, Boston University School of Education, 332 Bay State Road, Boston, Mass.

Fees

The registration fee for the entire period of the Institute is $2.00. The registration fee for members of The National Council, members of The Mathematical Association of America, teachers in elementary schools, and students, who attend from August 21 to 24 is $0.50. The fee for all others for the same period is $1.50. These fees include admission to all meetings within the time limits set except for the laboratories where an additional $1.50 is necessary to cover the cost of materials. No registration fee will be charged for members of a family who do not attend the meetings.

The inclusive rate for board, room, and operating expenses is $6.00 per day per person. There will be no extra charges for the banquets, tips, etc. There will be no refunds for parts of a day. A day begins with the evening meal and ends with the noon meal.

Reservations

Fill out the form on page 316 and mail it before July 1 to

H. Gray Funkhouser, Cilley Hall, Exeter, New Hampshire

Early registration is advised because the facilities are limited to 500 people. To hold a reservation it is necessary to include with it an advance payment of $5.00 per person. Make checks payable to The Association of Teachers of Mathematics in New England.

APPLICATION FOR RESERVATION

Check the items applicable to you:

Please reserve room (single ______, double ______, connecting singles ______) and board for August 21–24 (three days) ______, August 21–28 (seven days) ______. If you checked double room or connecting singles above, give name of person sharing ______________

Please reserve room and board for the following members of my family: ______________

____

Name (Print) ______________________________

Address for mail ______________________________

____

Position ______________________________

Member NCTM ______; member MAA ______; elementary school teacher ______; student ______ exhibitor ______.

PLANNING A NEW YEARBOOK—HOW YOU CAN HELP
AN ANNOUNCEMENT BY THE YEARBOOK PLANNING COMMITTEE

Upon recommendation by the Yearbook Planning Committee, the Board of Directors of the National Council of Teachers of Mathematics has voted to publish a yearbook on the subject of EMERGING PRACTICES IN MATHEMATICS EDUCATION. In the preparation of the yearbook, the Committee will be assisted by Dr. John R. Clark of Teachers College, Columbia University, a former editor of The Mathematics Teacher.

Emerging practices, whether in the elementary school, secondary school, or teacher education, are innovations which challenge the so-called standard, conventional approved practices. Emerging practices may or may not obtain wide acceptance in the near future. The Committee believes, however, that teachers generally are vitally interested in knowing about the work of those teachers who are experimenting with new subject-matter, new organizations of old subject matter, new teaching procedures, new techniques of evaluation, new approaches in teacher education (pre-service or in-service), new aids to learning, or new theories of learning.

We invite you to help us find teachers who are employing new, promising approaches to problems in mathematics education.

The following is a partial list of practices or areas of experimentation which have been suggested to the Yearbook Planning Committee as being appropriate for the yearbook: (1) Mathematics in the core program; (2) A four-year non-academic curriculum for the high school; (3) Approximate computation in the 7th and 8th grades; (4) A background course in arithmetic for teachers in elementary schools; (5) New approaches to long division; (6) Non-geometric originals in demonstrative geometry—more or fewer?; (7) A concept approach to algebra; (8) Mental arithmetic in the elementary school; (9) An arithmetic clinic; (10) Sub-grouping within a class; (11) The mathematics curriculum of a small high school; (12) Teacher and pupil-made learning aids; (13) New proposals for organizing the content of academic courses in mathematics; (14) Experience with concept tests in arithmetic; (15) Using "Group-process" in teaching mathematics; (16) A self-selected curriculum in arithmetic; (17) A workshop institute for teachers of mathematics; (18) A high school course in arithmetic; (19) Using history of mathematics in teaching secondary school mathematics; (20) Statistics needed by the high school pupils; (21) Using films and filmstrips in teaching; (22) Teaching space-perception—a new approach; (23) Choosing applications to fit pupil needs.

This list of topics is included to suggest the wide range and variety of the aspects of mathematics education. To date, no topics or contributors have been selected.

The Yearbook Planning Committee urges the members of the Council to cooperate in identifying potential contributors to the yearbook. On a two-cent postcard, addressed to John R. Clark, Teachers College, Columbia University, New York 27, N. Y., comment upon any practice (giving the name and address of the practitioner) which you think worthy of consideration for inclusion in the yearbook. PLEASE DO IT NOW.

Yearbook Planning Committee: Frank Lankford
Daniel Snader
F. Lynwood Wren
Veryl Schult, Chairman